

Firm Valuation: Free Cash Flow or Cash Flow to Equity?

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## Firm Valuation: Free Cash Flow or Cash Flow to Equity?

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### **Abstract**

In a M & M world, the equity value is the present value of the Free Cash Flow FCF at the Weighted Average Cost of Capital WACC minus debt and it should be identical to the present value of the CFE (cash flow to equity) discounted at the cost of equity capital,  $e$ . In this paper the relationship between firm value calculated through the FCF and the CFE is examined. Several approaches to the firm value calculations are presented. We present a complex model where we compare the results obtained with the traditional M&M WACC found in the literature with the WACC approach presented by Harris and Pringle 1985. They make two assumptions that differ from the traditional M & M WACC. First, they assume that the discount rate for the tax shield is  $\rho$ , the return to unlevered equity. Second, based on the first assumption, the expression for  $e$ , the return to levered equity does not include the factor  $(1-T)$ . Unlike the traditional WACC, the new WACC gives consistent results. However, the new WACC does not work if there are losses carried forward and/or the taxes are paid the following year. To solve these two practical issues, we present a new adjusted WACC. We call this adjusted WACC, TV WACC. In addition to the above-mentioned assumptions, this WACC is defined as a function of the real tax savings TS, earned. When this new approach is used, total consistency is found. In particular, with this new approach losses carried forward LCF and taxes paid at a different date as accrued, are taken into account and we obtain consistent values.

### **Keywords**

*Firm valuation, NPV, Free Cash Flow, FCF, Cash Flow to Equity, CFE, Cash Flow to Debt, CFD, Discounted Cash Flow, DFC.*

*JEL Classification: D92, E22, G12, G31, M40, M41, M46*

## **Introduction**

In a M & M world, the equity value is the present value of the Free Cash Flow FCF at the Weighted Average Cost of Capital WACC minus debt and it should be identical to the present value of the CFE (cash flow to equity) discounted at the cost of equity capital,  $e$ . In this paper the relationship between firm value calculated through the FCF and the CFE is examined. Several approaches to the firm value calculations are presented. We present a complex model where we compare the results obtained with the traditional M&M WACC found in the literature with the WACC approach presented by Harris and Pringle 1985. They make two assumptions that differ from the traditional M & M WACC. First, they assume that the discount rate for the tax shield is  $\rho$ , the return to unlevered equity. Second, based on the first assumption, the expression for  $e$ , the return to levered equity does not include the factor  $(1-T)$ . Unlike the traditional WACC, the new WACC gives consistent results. However, the new WACC does not work if there are losses carried forward and/or the taxes are paid the following year. To solve these two practical issues, we present a new adjusted WACC. We call this adjusted WACC, TV WACC. In addition to the above-mentioned assumptions, this WACC is defined as a function of the real tax savings TS, earned. When this new approach is used, total consistency is found. In particular, with this new approach losses carried forward LCF and taxes paid at a different date as accrued, are taken into account and we obtain consistent values.

### **Where value lies?**

Some people think that the value of a firm is found in the financial statements. Value will never be found in accounting statements. Value arises from expectations. And where are these expectations? They are in the future cash flows.

To understand these ideas assume that there is a box with a machine inside. Anyone could buy it, if she pays the price listed on the price tag. However, if that machine is accompanied by a strategy, a plan and a team led by an outstanding manager, it has the capacity to create wealth and value, greater than its price.

In fact, if somebody asks the owner of that project not to develop it, she will ask for a premium in order not to start the project. When valuing a firm the idea is to measure that value. And this value is based on *expectations* of what the machine and the team could do in terms of wealth creation.

The problems arise because what is sold is *the value that can be created in the future*. Just remember the boom of the *dot com* firms. Not even a dollar was earned and they sold the stocks for skyrocketed prices. What were they buying? Just value expectations.

Discounting the free cash flow FCF, for the firm at the Weighted Average Cost of Capital and subtracting the debt or discounting the cash flow to equity holders CFE, at the cost of equity, measures this future value. In a M & M world, these two values must be equal. However, current practice find that this does not happens. These two values are not equal.

This paper studies this problem. The problem lies in what is the correct discount rate to be used to discount the cash flows. We will show how using different approaches we arrive at different figures and we will show approaches that produce consistent results and those that do not.

### ***Some background on WACC***

In the literature we find the traditional presentation for WACC for discounting the FCF excluding tax savings from interest payments. It is expressed as

$$\text{WACC} = eE\% + d(1-T)D\% \quad (1)$$

Where e is the cost of equity, E% is the proportion of equity in the total value, d is the cost of debt before taxes, D% is the proportion of debt in total value and T is tax rate.

In the traditional presentation, e is calculated as

$$e = \rho + (1-T) (\rho - d)D\%/E\% \quad (2)$$

And the discount rate for the tax shield is d. We will call this the MM WACC.

We assume that there are no losses and there are no losses carried forward (LCF) and taxes are paid the same year as accrued. Unfortunately, this traditional formulation does not work in the finite period cash flows.

We will examine the WACC proposed by Harris and Pringle (1985) and we call it the HP WACC. This WACC assumes that there are no losses and there are no losses carried forward (LCF) and taxes are paid the same year as accrued. This HP WACC differs from the MM WACC in two significant ways. First, the discount rate for the tax shield is  $\rho$ . Second, based on the first assumption, the expression for e does not include (1-T). This is,

$$e = \rho + (\rho - d)D\%/E\% \quad (3)$$

In practice we encounter losses carried forward (LCF). If there are LCF neither of the two expressions for WACC works. Then the correct WACC proposed by Tham & Velez (TV) is

$$\text{TV WACC} = \rho - \text{TS}/(\text{Total levered value}) \quad (4)$$

This last version of WACC assumes  $\rho$  as the discount rate for the Tax savings. It can be seen as a different presentation of the typical WACC formula. In line 4, the TS refers to the difference in the tax shield between the levered cash flows and the unlevered cash flows.

$$\text{WACC} = eE\% + dD\% - TdD\% \quad (5)$$

$$\text{WACC} = eE\% + dD\% - TdD/(\text{Total levered value}) \quad (6)$$

$$\text{WACC} = eE\% + dD\% - TS/(\text{Total levered value}) \quad (7)$$

$$\text{WACC} = \rho - TS/(\text{Total levered value}) \quad (8)$$

This presentation is the most general way to express WACC. It has the feature that considers tax savings when they are in fact earned. And this means that losses carried forward are included in the analysis.

In this paper we will show a comparison between the results obtained with the three versions for WACC<sup>1</sup>.

### ***A Complex Model***

All these approaches work with a complex model. The complex model has several modules (See Annex).

In tables A1 to A7 are the inputs. With these inputs we calculate interest rates (from real rates, inflation, risk premium and debt ratio (accounting debt ratio)) for each year. The inputs include initial costs, equity investments, accounts receivable and payable policies, a cash cushion<sup>2</sup>, dividend and inventory “policies”, volume increases, prices, prices increases and an elasticity function among others. This information is used to calculate firm value and NPV in order to compare with market values.

Tables A8 to A11 present the forecasts of every variable including units sold, prices and the rest. From tables A12 to A16, the financial statements: Balance Sheet, P&L statements and Cash Budget or Working Cash Statement. These financial statements are all linked. There is not what some authors call “plugs” (an account where any difference between total assets and liabilities plus equity are included in order the Balance Sheet checks).

In reality, the “plug” is a decision: either to borrow money or to invest it (or leave it in the bank). In the Cash Budget you will find that when the balance is in red, automatically it determines how much to borrow

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<sup>1</sup> We are restricting our comparison to the traditional MM WACC. We recognize that many other WACC are found in the literature. See Taggart 1991 and Fernandez, 2000

<sup>2</sup> It is assumed that this amount of cash is deposited in a check account that is a non-bearing interest account. As it is invested in the firm, it is expected to earn at least the WACC.

taking into account the cash cushion. In the cash budget the dividends paid are included.

Table 17 shows the expected WACC and growth beyond year 4.

Tables A18 and A19 present the cash flow construction. The FCF, CFD and the CFE. As can be observed, the FCF is constructed from the Cash Budget balance and deduct all the items related with the financing: loans received and paid, interest paid, equity from stockholders, dividends and tax savings.

In this model there exists some options:

1. To make inflation adjustments
2. To pay taxes the same year as accrued or not.
3. To reinvest cash surpluses.

These options are handled with a dummy variable. Not all the options are used in the example presented in the Annex.

Another features of the model are the presence of an elasticity factor that relates price increase and demand. The possibility to consider inventory, accounts receivable and payable policies, payout ratios, cash cushion, etc. is included.

All this means that it is not necessary to consider changes in working capital as with other methods. It is calculated just as it is expected it will be in the Cash Budget.

With this information, we calculate in the last part, Tables A20 to A33, the WACC and firm value. These calculations are done fixing  $\rho$  for year 0. We present the value calculations for the three WACC approaches presented above.

A very important feature is the losses carried forward LCF. This is quite important because this means that tax savings are earned when taxes are paid. When tax savings are not earned (for instance, when there are losses) and losses carried forward, the analysis with WACC is inconsistent.  $PV(FCF) = PV(CFE) + PV(CFD)$  does not hold, unless you use  $d(1-T)$  as the cost of debt after taxes. It doesn't matches if the cost of debt after taxes is calculated from CFD after taxes or if taxes are paid next year. These inconsistencies are solved using the Adjusted WACC or TV WACC =  $\rho - TS_t / (\text{Total value at } t-1)$ . When Adjusted WACC is used, consistency is obtained.

In the complex model, the calculations of firm value and the different WACC's are done when there are losses and LCF and when there are no losses. In all cases it is assumed that taxes are paid at the same year as accrued. The cost of debt after taxes is calculated as the IRR for the CFD.

The results for the first case are summarized in Table 1 Panels A and B, where we assume that there are losses, the losses are carried forward and taxes are paid the same year.

The present value of the FCF at the TV WACC (total market value) is \$47,176.3; with the HP WACC, the present value is \$47,291.5, which is higher than the previous value by 115.1. The reason the present value with the HP WACC is higher is because the difference in tax savings due to the LCF. When calculating the Adjusted Present Value APV, there is a difference of 1,312.9, for the same reason.

The present value of the FCF at the TV WACC (total market value) is \$47,176.3; with the traditional MM WACC, the present value is \$48,225.4, which is higher than the previous value by 1,996.4. The reason the present value with the MM WACC is higher is because the discount rate for the tax shield is  $d$  lower than  $\rho$ . This means a lower WACC. Another reason is the difference in tax savings due to the LCF. When calculating the Adjusted Present Value APV, there is a difference of 253.2, for the same reasons.

There are also differences when calculating the market equity value. The differences are the same when calculated as Total value less debt. However, when the market equity value is calculated from the CFE at  $e$ , there differences. The present value of CFE at  $e$  under TV WACC assumptions is 31,066.3; with the HP WACC, the present value is \$32,316.4, which is higher than the previous value by 1,250.1. The reason the present value with the HP WACC is higher is because the difference in tax savings due to the LCF. With the traditional MM WACC, the present value is \$33,275.7, which is higher than the previous value by 2,765.3. The reasons for this difference are the lower value for  $e$  (due to the  $(1-T)$  factor) and the difference in tax savings due to the LCF.

The results for the second case are summarized in Table 2 Panels A and B, where we assume that there are no losses and taxes are paid the same year.

The present value of the FCF at the TV WACC (total market value) is \$47,505.9; with the HP WACC, the present value is the same. When calculating the Adjusted Present Value APV, there is no difference.

The present value of the FCF at the TV WACC (total market value) is \$47,505.9; with the traditional MM WACC, the present value is \$48,439.8, which is higher than the previous value by 933.9. The reason the present value with the MM WACC is higher is because the discount rate for the tax shield is  $d$ , lower than  $\rho$ . This means a lower WACC. When calculating the Adjusted Present Value APV, there is a difference of 376.8, for the same reason.

There are also differences when calculating the market equity value. The differences are the same when calculated as Total value less debt. However, when the market equity value is calculated from the CFE at  $e$ ,

there differences. The present value of CFE at  $e$  under TV WACC assumptions is 31,395.9; with the HP WACC, the present value is the same. With the traditional MM WACC, the present value is \$32,329.8, which is higher than the previous value by 933.9. The reasons for this difference are the lower value for  $e$  (due to the  $(1-T)$  factor) and the difference in tax savings due to the LCF.

In summary

Table 1 Panel A. Losses, losses carried forward and taxes paid the same year Total market value.

Value calculation (1)	With TV WACC (TV WACC = $\rho - TS / (\text{Total value})$ $e = \rho + (\rho - d)D/E$ (2)	With HP WACC $e = \rho + (\rho - d)D/E$ (3)	Difference with TV WACC (4)	% over TV WACC (5)	With MM WACC $e =$ $\rho + (1 - T)(\rho - d)D/E$ (6)	Difference with TV WACC (7)	% over TV WACC (8)
Total value PV(FCF @ WACC)	47,176.3	47,291.5	115.1	0.2%	48,225.4	1,996.4	2.2%
Total value PV(CCF=FCF + TS @ $\rho$ )	47,176.3	48,489.2	1,312.9	2.8%			
Total value = PV(FCF at $\rho$ ) + PV(TS at d)					47,429.5	253.2	0.5%

Table 1 Panel B. Losses, losses carried forward and taxes paid the same year Total market value. Equity market value

Value calculation (1)	With TV WACC (TV WACC = $\rho -$ $TS / (\text{Total value})$ $e = \rho + (\rho - d)D/E$ (2)	With HP WACC $e = \rho + (\rho - d)D/E$ (3)	Difference with TV WACC (4)	% over TV WACC (5)	With MM WACC $e =$ $\rho + (1 - T)(\rho - d)D/E$ (6)	Difference with TV WACC (7)	% over TV WACC (8)
Equity value = Total value (from FCF at WACC) - debt	31,066.3	31,181.5	115.1	0.4%	32,115.4	1,996.4	3.4%
Equity value = Total value (from CCF at $\rho$ ) - debt	31,066.3	32,379.2	1,312.9	4.2%			
Equity value = PV(FCF at $\rho$ ) + PV(TS at d) - debt					31,319.5	253.2	0.8%
Equity value = PV(CFE @ e)	31,066.3	32,316.4	1,250.1	4.0%	33,275.7	2,765.3	7.1%

Column (1) of Table 1 (panels A and B) shows the name of the procedure to calculate Total or Equity value. Columns (2) and (3) show the different values when the Adjusted or TV WACC =  $\rho - TS / (\text{Total value})$  and HP WACC =  $eE\% + d(1-T)D\%$ . Both of them consider  $e = \rho + (\rho - d)D/E$  and assume  $\rho$  as the correct discount rate for the tax savings, TS. Columns (4) and (5) show the absolute and relative difference with the TV WACC. From columns (4) and (5) it can be seen that in this case TV WACC and HP WACC differ. Column (6) shows the values when calculated with the MM WACC, this is WACC =  $eE\% + d(1-T)D\%$  where  $e = \rho + (1-T)(\rho - d)D/E$  Columns (7) and (8) show the absolute and relative difference with the TV WACC. In table 2 the traditional WACC (MM WACC) also shows inconsistencies among the different values and between those values and the TV WACC. Notice the internal consistency of TV WACC. The very small differences between TV WACC and HP WACC might show that for practical purposes HP WACC is a good approximation. However, if precision is desired, TV WACC should be used.

Table 2 Panel A No losses, taxes paid the same year

Value calculation (1)	With TV WACC (TV WACC = $\rho - \text{TS}/(\text{Total value})$ $e = \rho + (\rho - d)D/E$ (2)	With HP WACC $e = \rho + (\rho - d)D/E$ (3)	Difference with TV WACC (4)	% over TV WACC (5)	With MM WACC $e = \rho + (1 - T)(\rho - d)D/E$ (6)	Difference with TV WACC (7)	% over TV WACC (8)
Total value = PV(FCF at WACC)	47,505.9	47,505.9	(0.0)	0.0%	48,439.8	933.9	2.0%
Total value = PV(CCF = FCF + TS at $\rho$ )	47,505.9	47,505.9	(0.0)	0.0%			
Total value = PV(FCF at $\rho$ ) + PV(TS at d)					47,882.7	376.8	0.8%

Table 2 Panel B. No losses, taxes paid the same year

Value calculation (1)	With TV WACC (TV WACC = $\rho - \text{TS}/(\text{Total value})$ $e = \rho + (\rho - d)D/E$ (2)	With HP WACC $e = \rho + (\rho - d)D/E$ (3)	Difference with TV WACC (4)	% over TV WACC (5)	With MM WACC $e = \rho + (1 - T)(\rho - d)D/E$ (6)	Difference with TV WACC (7)	% over TV WACC (8)
Equity value = Total value (from FCF at WACC) - debt	31,395.9	31,395.9	(0.0)	0.0%	32,329.8	933.9	3.0%
Equity value = Total value (from CCF at $\rho$ ) - debt	31,395.9	31,395.9	(0.0)	0.0%			
Equity value = PV(FCF at $\rho$ ) + PV(TS at d) - debt					31,772.7	376.84	1.2%
Equity value = PV(CFE at e)	31,395.9	31,395.9	0.0	0.0%	32,329.8	933.9	3.0%

Column (1) of Table 2, Panels A and B, shows the name of the procedure to calculate Total or Equity value. Columns (2) and (3) show the different values when the Adjusted or TV WACC =  $\rho - \text{TS}/(\text{Total value})$  and TV WACC =  $eE\% + d(1 - T)D\%$  are used. Both of them consider  $e = \rho + (\rho - d)D/E$  and assume  $\rho$  as the correct discount rate for the tax savings, TS. Columns (4) and (5) show the absolute and relative difference with the TV WACC. In Table 1, No losses, taxes paid the same year and it is shown that there is no difference. Column (6) shows the values when calculated with the MM WACC, this is WACC =  $eE\% + d(1 - T)D\%$  where  $e = \rho + (1 - T)(\rho - d)D/E$ . Columns (7) and (8) show the absolute and relative difference with the TV WACC. In table 2 the traditional WACC (MM WACC) shows inconsistencies among the different values and between those values and the TV WACC. Notice the internal consistency of TV WACC and HP WACC.

Table 3 No losses, taxes paid the same year. Market and book values ratios

	Year 1	Year 2	Year 3	Year 4
Book value for equity	24,000.0	24,050.7	26,378.6	30,961.8
Market value for equity	31,395.9	43,022.3	55,149.1	67,693.5
Market equity value/equity book value	1.3	1.8	2.1	2.2
Book value for total assets	40,110.0	39,125.1	37,809.2	38,773.9
Market total value	47,505.9	55,104.8	63,204.1	71,721.0
Market total value/Book value for total assets	1.2	1.4	1.7	1.8

Table 4. Losses, losses carried forward and taxes paid the same year. Market and book values ratios

	Year 1	Year 2	Year 3	Year 4
Book value for equity	24,000.0	23,953.7	26,221.5	30,719.8
Market value for equity	31,066.3	42,651.4	54,708.3	67,193.1
Market equity value/equity book value	1.3	1.8	2.1	2.2
Book value for total assets	40,110.0	39,030.4	37,654.8	38,534.7
Market total value	47,176.3	54,733.9	62,763.3	71,220.6
Market total value/Book value for total assets	1.2	1.4	1.7	1.8

Tables 3 and 4 show the differences in market values when compared with book values. The equity market values are taken from the calculations made with the TV WACC approach. We consider that this is the correct method. Although not a formal proof, the tables give an idea on how much difference is present when the book values are used. These differences are explained because the financial statements do not consider any terminal value. Frequently, terminal value accounts for more than 50% of the total value of the firm.

### **Conclusions**

We have shown three approaches to calculate total and equity value with different expressions for WACC. They are the MM WACC (the traditional WACC), the HP WACC and the TV WACC. The first one produces inconsistent results. The second one is consistent as long as there are no losses and/or losses carried forward. The last one, the TV WACC, produces consistent results either with no losses or losses and losses carried forward. It can be shown that when taxes are paid the following year after accrual, the only one that gives consistent results is TV WACC.

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ANNEX

**FINANCIAL ASSESMENT FOR THE CREATION OF A FIRM  
LOSSES AND LOSSES CARRIED FORWARD (LCF)**

Table A1 External or macroeconomic variables

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax rate		40.0%	40.0%	40.0%	40.0%
Inflation rate		12.00%	11.00%	10.00%	9.00%

This information can be obtained from macroeconomic forecasts made by the government or associations or universities.

Table A2 Market variables

	Year 0	Year 1	Year 2	Year 3	Year 4
Quantity to sell at year t at price given at year 0	7,247				
Increase in volume		0.00%	7.00%	5.00%	4.00%

The quantity and increase in the market might be obtained by a marketing research.

Table A3 Risk free discount rate and risk premium estimates

	Year 0	Year 1	Year 2	Year 3	Year 4
Real risk free discount rate		6.00%	6.00%	6.00%	6.00%
Rate of interest for debt					
Risk premium for debt		10.00%	10.00%	10.00%	10.00%
Effective rate before taxes		30.59%	30.59%	30.59%	30.59%
Annual nominal rate compounded twice a year		28.55%	28.55%	28.55%	28.55%
Effective rate after taxes		22.62%	22.62%	22.62%	22.62%

This information can be estimated based on the behavior of bonds or simply setting the cost of debt from a contract between the firm and the debt holder. Fisher Theorem is used to compose the three components.

Table A4 Price and increase in price

	Year 0	Year 1	Year 2	Year 3	Year 4
Purchase price	\$2.95				
Real price increase		0.90%	1.64%	1.00%	0.00%
Annual increase in purchase price		13.01%	12.82%	11.10%	9.00%

The purchase price is a function of Q, the estimated quantity the market will buy.

Table A5 Internal variables (endogenous) with some degree of negotiability

	Year 0	Year 1	Year 2	Year 3	Year 4
Selling price	\$5.60				
Real price increase		0.10%	0.60%	0.80%	0.70%
Increase in selling price		12.11%	11.67%	10.88%	9.76%
Elasticity factor. Effect of price change upon demand. (1-.366*((1+increase in selling price)/(1+inflation)-1))		0.9996	0.9978	0.9971	0.9974

The elasticity function has to be found for the specific product or service we are dealing with. It is a function of selling price increase. In the example, an elasticity function calculated for the domestic water supply was used. Selling price is the result of a market survey and/or the cost structure of the good or service.

Table A6 Costs

	Year 0	Year 1	Year 2	Year 3	Year 4
Monthly auditing fees	\$37.00	\$0.00	\$0.00	\$0.00	\$0.00
Real price increase		1.000%	1.30%	1.30%	0.90%
Increase in monthly auditing fees		13.12%	12.44%	11.43%	9.98%
Monthly overhead	\$145.00	\$0.00	\$0.00	\$0.00	\$0.00
Real price increase		0.900%	1.10%	1.20%	1.10%
Increase in overhead		13.01%	12.22%	11.32%	10.20%
Monthly payroll					
Administrative payroll	\$160.00	\$0.00	\$0.00	\$0.00	\$0.00
Monthly payroll of selling force	\$40.00	\$0.00	\$0.00	\$0.00	\$0.00
Real price increase		1.20%	1.40%	1.20%	0.60%
Increase in payroll		13.34%	12.55%	11.32%	9.65%
Sales commissions		3.00%	3.00%	3.00%	3.00%
Fringe benefits and taxes over payroll	56.93%	\$0.00	\$0.00	\$0.00	\$0.00

These are stated on a monthly base. Price increases might be estimated based on historical data of similar firms, or simply a subjective forecast.

Table A7 Targets and/or operating policies

	Year 0	Year 1	Year 2	Year 3	Year 4
Advertising and promotions (% on sales)		3.00%	3.00%	3.00%	3.00%
Equity contribution	24,000.00				
Fixed assets	40,000.00				
Depreciation (lineal for 5 years)					
Final inventory as percent of sales (units)		8.33%	8.33%	8.33%	8.33%
Inventories valued at FIFO					
Percentage of sales received in the same year		95.00%	95.00%	95.00%	95.00%
Percentage of sales received in the next year		5.00%	5.00%	5.00%	5.00%
Percentage of payments made the same year as accrued (overhead and suppliers)		90.00%	90.00%	90.00%	90.00%
Percentage of payments made the next year as accrued (overhead and suppliers)		10.00%	10.00%	10.00%	10.00%
Percentage of net profits (dividends) paid to stockholders the year after obtained the net profits		30.00%	30.00%	30.00%	30.00%
Minimum cash balance desired after financing deficits	110	120	131	160	160

Many variables are just policies or target the firm will pursue.

Table A8 Forecasts based on Tables A1 to A7

	Year 1	Year 2	Year 3	Year 4
Units sold (increase plus elasticity effect) S	7,243.90	7,733.95	8,096.87	8,399.17
Final inventory in units FI	603.66	644.50	674.74	699.93
Initial inventory in units II	0.00	603.66	644.50	674.74
Purchases in units $P = S + FI - II$	7,847.56	7,774.79	8,127.11	8,424.36

Units to be sold are based on the initial market survey, the increase in volume and the elasticity factor for the product or service.

Table A9 Depreciation

	Year 1	Year 2	Year 3	Year 4
Annual depreciation	8,000.00	8,000.00	8,000.00	8,000.00
Cumulative depreciation	8,000.00	16,000.00	24,000.00	32,000.00

This depreciation is based on the lineal method stated above.

Table A10 Selling and purchasing prices

	Year 1	Year 2	Year 3	Year 4
Selling price	6.28	7.01	7.77	8.53
Buying price	3.13	3.56	3.81	4.03

These prices are based on the initial selling and purchasing price and the price increase stated above. The nominal selling price increase defines the elasticity factor.

Table A11 Administrative and selling expenses

Annual expenses	Year 1	Year 2	Year 3	Year 4
Administrative payroll	2,176.20	2,449.41	2,726.68	2,989.91
Administrative fringe benefits on payroll	181.35	204.12	227.22	249.16
Sales force payroll	544.05	612.35	681.67	747.48
Sales commissions	1,364.37	1,626.61	1,888.22	2,149.95
Fringe benefits (annual premium 1/12 of payroll)	159.04	186.58	214.16	241.45
Other fringe benefits (46,67% payroll and commissions)	1,985.14	2,278.56	2,574.15	2,861.26
Auditing fees	502.25	564.75	629.30	692.11
Overhead	1,966.34	2,206.65	2,456.44	2,706.97
Advertising and promotions	1,364.37	1,626.61	1,888.22	2,149.95

These costs are based on the monthly based costs above. In this table they are annualized.

Table A12 Balance sheet

	Year 0	Year 1	Year 2	Year 3	Year 4
Assets					
Cash	110.00	2,866.28	8,650.06	16,813.62	27,312.21
Accounts receivable		2,273.96	2,711.02	3,147.0	3,583.25
Inventory		1,890.20	2,293.68	2,574.04	2,823.88
Fixed assets	40,000.00	40,000.00	40,000.00	40,000.00	40,000.00
Cumulative depreciation		8,000.00	16,000.00	24,000.00	32,000.00
Net fixed assets	40,000.00	32,000.00	24,000.00	16,000.00	8,000.00
Total	40,110.00	39,030.43	37,654.76	38,534.69	41,719.33
Liabilities and equity					
Accounts payable (suppliers)		2,457.26	2,766.94	3,100.39	3,398.81
Accounts payable (overhead)		196.63	220.66	245.64	270.70
Fringe benefits accrued		340.39	390.70	441.38	490.61
Interest charges accrued					
Accrued taxes		0.00	0.00	0.00	0.00
Long term debt	16,110.00	12,082.50	8,055.00	4,027.50	0.00
Total liabilities	16,110.00	15,076.78	11,433.31	7,814.91	4,160.12
Equity	24,000.00	24,000.00	24,000.00	24,000.00	24,000.00
Cumulative retained earnings			(46.34)	1,541.11	5,166.18
Net income for the year		(46.34)	2,267.80	5,178.67	8,393.03
Total net equity	24,000.00	23,953.66	26,221.45	30,719.78	37,559.21
Total	40,110.00	39,030.43	37,654.76	38,534.69	41,719.33

All the financial statements are linked. For instance, cash is the same value found in the Cash Budget as cumulative cash balance. Net income

for the year is the same found in the P&L statement, and so on. There is no plug.

Table A13 Profit and losses statement

	Year 1	Year 2	Year 3	Year 4
Sales	45,479.16	54,220.37	62,940.69	71,664.93
Cost of goods sold	22,682.38	27,265.97	30,723.52	33,738.31
Initial inventory	0.00	1,890.20	2,293.68	2,574.04
Purchases	24,572.57	27,669.45	31,003.89	33,988.15
Final inventory	1,890.20	2,293.68	2,574.04	2,823.88
Gross profit	22,796.79	26,954.40	32,217.16	37,926.62
Selling and administrative expenses	18,243.13	19,755.63	21,286.05	22,788.24
Payroll expenses	4,705.40	5,340.32	5,982.50	6,598.65
Annual premium	340.39	390.70	441.38	490.61
Auditing fees	502.25	564.75	629.30	692.11
Selling commissions	1,364.37	1,626.61	1,888.22	2,149.95
Overhead	1,966.34	2,206.65	2,456.44	2,706.97
Depreciation	8,000.00	8,000.00	8,000.00	8,000.00
Advertising	1,364.37	1,626.61	1,888.22	2,149.95
Earnings before interest and taxes	4,553.66	7,198.77	10,931.11	15,138.38
Other expenses (interest expenses)	4,600.00	3,450.00	2,300.00	1,150.00
Earnings before taxes	(46.34)	3,748.76	8,631.11	13,988.38
Taxes	0.00	1,480.97	3,452.44	5,595.35
Net profit	(46.34)	2,267.80	5,178.67	8,393.03

The figures for the P&L statements are based on the forecasts above. Sales are simply quantity to be sold times the rice, etc. Items such as taxes will appear in the Balance Sheet in case taxes are paid the following year. In this case, they appear as a cash outflow at the Cash Budget.

Table A14 Cash budget: Cash inflows

Cash inflows		Year 0	Year 1	Year 2	Year 3	Year 4
Cash collection of sales year 1	45,479.16		43,205.20	2,273.96		
Cash collection of sales year 2	54,220.37			51,509.35	2,711.02	
Cash collection of sales year 3	62,940.69				59,793.65	3,147.03
Cash collection of sales year 4	71,664.93					68,081.69
Total income accounts receivable			43,205.20	53,783.31	62,504.67	71,228.72
Equity in cash		24,000.00				
Total cash inflows		24,000.00	43,205.20	53,783.31	62,504.67	71,228.72

In this table all the cash inflows are registered. It takes into account the accounts receivables policy stated above. It does not include loans income. It is a financing decision that will be made with the cash balance below.

Table A15 Cash budget: Cash outflows

Cash outflows		Year 0	Year 1	Year 2	Year 3	Year 4
Suppliers						
Payments for purchases year 1	24,572.57		22,115.32	2,457.26		
Payments for purchases year 2	27,669.45			24,902.50	2,766.94	
Payments for purchases year 3	31,003.89				27,903.50	3,100.39
Payments for purchases year 4	33,988.15					30,589.33
Total payments to suppliers			22,115.32	27,359.76	30,670.44	33,689.72
Payroll and fringe benefits			4,705.40	5,340.32	5,982.50	6,598.65
Annual premium year				340.39	390.70	441.38
Auditing fees			502.25	564.75	629.30	692.11
Commissions on sales			1,364.37	1,626.61	1,888.22	2,149.95
Payment of overhead year 1	1,966.34		1,769.71	196.63		
Payment of overhead year 2	2,206.65			1,985.98	220.66	
Payment of overhead year 3	2,456.44				2,210.79	245.64
Payment of overhead year 4	2,706.97					2,436.27
Advertising			1,364.37	1,626.61	1,888.22	2,149.95
Purchase of fixed assets		40,000.00	0.00	0.00	0.00	0.00
Interest expenses			4,600.00	3,450.00	2,300.00	1,150.00
Dividend payments				0.00	680.34	1,553.60
Taxes			0.00	1,480.97	3,452.44	5,595.35
Total cash outflows		40,000.00	36,421.43	43,972.02	50,313.62	56,702.63

This table shows all the cash outflows. It does not include loans payment. These are included below.

Table A16 Financing decision

	Year 0	Year 1	Year 2	Year 3	Year 4
Total cash inflows	24,000.00	43,205.20	53,783.31	62,504.67	71,228.72
Total cash outflows	40,000.00	36,421.43	43,972.02	50,313.62	56,702.63
Net cash gain (loss)	(16,000.00)	6,783.78	9,811.29	12,191.05	14,526.1
Cash balance at end of year	(16,000.00)	6,893.78	12,677.56	20,841.12	31,339.71
Bank loans	16,110.00	0.00	0.00	0.00	0.00
Repayment of loans (principal)		4,027.50	4,027.50	4,027.50	4,027.50
Investment of surplus	0.00	0.00	0.00	0.00	
Net cash gain (loss) after financing	110.00	2,756.28	5,783.79	8,163.55	10,498.59
Cash balance at end of year after financing and reinvestment	110.00	2,866.28	8,650.06	16,813.62	27,312.21

This table shows the information from the Cash Budget in order to make the financing decisions. These are, how much should be borrowed and how much can be paid with the available cash.

Table 17 Discount and growth rate beyond year 4

Discount rate for cash flows beyond year 4	23.11%
Growth for year 5	10.10%
Growth for year 6 and following years	0.00%

This table shows the growth and discount rate for determining the terminal value.

Table A18 Construction of Free Cash Flow (FCF) from the Net Cash Gain at the Cash Budget

		Year 1	Year 2	Year 3	Year 4	Year 5
Net cash gain (loss) after financing	(Plus)	2,756.28	5,783.79	8,163.55	10,498.59	
Bank loans	(Minus)	0.00	0.00	0.00	0.00	
Repayment of loans (principal)	(Plus)	4,027.50	4,027.50	4,027.50	4,027.50	
Interest expenses	(Plus)	4,600.00	3,450.00	2,300.00	1,150.00	
Tax shield for interest payments	(Minus)	0.00	(1,380.00)	(920.00)	(460.00)	0.00
Dividends	(Plus)	0.00	0.00	680.34	1,553.60	
Equity in kind	(Minus)	-	-	-	-	
Equity in cash	(Minus)	0.00	0.00	0.00	0.00	
Cash flow without terminal value		11,383.78	11,881.29	14,251.39	16,769.69	
FCF <sub>n+1</sub> = (FCF <sub>n</sub> (1+Growth rate for n+1)						18,464.13
Terminal value	(Plus)				79,912.36	
Free cash flow after taxes		11,383.78	11,881.29	14,251.39	96,682.05	

The FCF is calculated from the Cash Budget. We do not use the approach to calculate FCF from EBIT or net profit because this method increases the probability of errors. In particular, starting from EBIT might introduce errors in the tax paid and the tax savings earned.

Table A19 Cash Flow to Equity CFE and Cash Flow to Debt

	Year 1	Year 2	Year 3	Year 4
Cash flow for stockholder (dividends plus any cash )	2,756.28	5,783.79	8,843.89	91,964.55
Financing cash flow (after tax) CFD	8,627.5	6,097.5	5,407.5	4,717.5
Cost of debt as IRR of after tax CFD	17.13%			
Financing cash flow (before tax)	8,627.50	7,477.50	6,327.50	5,177.50
Cost of debt as IRR of before tax CFD	28.55%			
Free cash flow (FCF)	11,383.78	11,881.29	14,251.39	96,682.05
Financing cash flow (CFD)	8,627.50	6,097.50	5,407.50	4,717.50
Stockholder's cash flow (CFE)	2,756.28	5,783.79	8,843.89	91,964.55
Check: CFD + CFE =FCF	11,383.78	11,881.29	14,251.39	96,682.05

This table shows the CFD, the CFE, the cost before and after taxes calculated as the IRR of the CFD before and after taxes. Cost of debt is assumed constant.

The next tables show the calculation of market values and the three different approaches to calculate WACC. First, we present the TV WACC approach.

Table A20 Basic information for TV WACC calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax rate		40.00%	40.00%	40.00%	40.00%
Cost of debt before taxes		28.55%	28.55%	28.55%	28.55%
Given nominal $\rho$ year 1 and assuming constant risk		40.15%	38.90%	37.65%	36.40%
Real $\rho$ (deflated)		25.13%	25.13%	25.13%	25.13%
Discount factor at $\rho$		0.7135	0.5137	0.3732	0.2736
FCF		11,383.78	11,881.29	14,251.39	96,682.05
Debt (balance)	16,110.00	12,082.50	8,055.00	4,027.50	
Initial Equity contribution (book value)	24,000.00				
Initial investment (fixed assets plus cash)	40,110.00				
Interest payments		4,600.00	3,450.00	2,300.00	1,150.00
Debt payment		4,027.50	4,027.50	4,027.50	4,027.50
Cash flow to debt before taxes		8,627.50	7,477.50	6,327.50	5,177.50
Tax savings (TS = TdD)		0	1,380.00	920.00	460.00

This information is common for the three methods, except for the tax savings TS. With the TV WACC approach, TS are calculated in an explicit way. This is, we determine how much TS is earned according to the financial situation of the firm. We do not calculate the TS straightforward with the formula TdD.

Table A21 TV WACC and market value calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
WACC after taxes = $\rho - TS/\text{Total value at } t-1$		40.15%	36.38%	36.18%	35.75%
Market Total Value at t @ WACC	47,176.34	54,733.85	62,763.30	71,220.61	

In table the resulting WACC and market values are shown. It has to be remembered that there exists circularity and this is the result of iterations done by the spreadsheet.

Table A22 Capital Cash Flow and market value calculations at  $\rho$

	Year 0	Year 1	Year 2	Year 3	Year 4
Capital cash flow (CCF = FCF + TS)		11,383.78	13,261.29	15,171.39	97,142.05
Total value	47,176.34	54,733.85	62,763.30	71,220.61	

Capital cash flow is defined as  $CCF = FCF + TS$ . In this table the market value is calculated directly with the CCF and  $\rho$ .

Table A23 APV and market value calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
PV(FCF at $\rho$ )	45,998.22	45,998.22	53,082.73	61,849.91	70,883.36
PV(TS at $\rho$ )	1,178.11	1,178.11	1,651.12	913.39	337.25
Total	47,176.34				

Capital cash flow and APV with the TS discounted at  $\rho$ , are the same.

Table 24 CFE,  $e$  and market value of equity calculation with CFE at  $e$

	Year 0	Year 1	Year 2	Year 3	Year 4
$e = \rho + (\rho - d)D/E$		46.16%	41.83%	38.99%	36.87%
$CFE = FCF + TS - CFD$		2,756.28	5,783.79	8,843.89	91,964.55
PV(CFE at $e$ )		31,066.34	42,651.35	54,708.30	67,193.11

In table the resulting  $e$  and equity market values are shown. It has to be remembered that there exists circularity and this is the result of iterations done by the spreadsheet.

In the following tables the HP WACC approach is presented,

Table A25 Basic information for HP WACC calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax rate		40.00%	40.00%	40.00%	40.00%
Cost of debt before taxes		28.55%	28.55%	28.55%	28.55%
Given nominal $\rho$ year 1 and assuming constant risk		40.15%	38.90%	37.65%	36.40%
Real $\rho$ (deflated)		25.13%	25.13%	25.13%	25.13%
Discount factor at $\rho$		0.7135	0.5137	0.3732	0.2736
FCF		11,383.78	11,881.29	14,251.39	96,682.05
Debt (balance)	16,110.00	12,082.50	8,055.00	4,027.50	
Initial Equity contribution (book value)	24,000.00				
Initial investment (fixed assets plus cash)	40,110.00				
Interest payments		4,600.00	3,450.00	2,300.00	1,150.00
Debt payment		4,027.50	4,027.50	4,027.50	4,027.50
Cash flow to debt before taxes		8,627.50	7,477.50	6,327.50	5,177.50
Tax savings (TS = TdD)		1,840.00	1,380.00	920.00	460.00

This information is common for the three methods, except for the tax savings TS. With the TV WACC approach, TS are calculated in an explicit

way. This is, we determine how much TS is earned according to the financial situation of the firm. In this case we calculate the TS straightforward with the formula TdD.

Table A26 HP WACC and market value calculation at HP WACC Year 0  $\rho$  known

	Year 0	Year 1	Year 2	Year 3	Year 4
Debt					
Weight for Debt D%		34.07%	22.40%	12.92%	5.67%
After tax cost of debt		22.62%	22.62%	22.62%	22.62%
Contribution of debt to WACC		7.71%	5.07%	2.92%	1.28%
Equity					
Weight for Equity E%		65.93%	77.60%	87.08%	94.33%
$e_t = \rho_t + (\rho_t - d)D_{t-1}/E_{t-1}$		46.14%	41.88%	39.00%	36.87%
Contribution of equity to WACC		30.42%	32.50%	33.96%	34.78%
WACC after taxes		38.1%	37.6%	36.9%	36.1%
Market Value at t @ WACC	47,291.47	53,939.94	62,324.11	71,058.45	

In table the resulting WACC and market values are shown. The HP WACC is calculated based on the contribution of each: the debt and the equity. It has to be remembered that there exists circularity and this is the result of iterations done by the spreadsheet.

Table A27 Capital Cash Flow and market value calculations at  $\rho$

	Year 0	Year 1	Year 2	Year 3	Year 4
Capital Cash Flow (CCF = FCF + TS)		13,223.78	13,261.29	15,171.39	97,142.05
Total value		48,489.22	54,733.85	62,763.30	71,220.61

Capital cash flow is defined as  $CCF = FCF + TS$ . In this table the market value is calculated directly with the CCF and  $\rho$ .

Table A28 APV and market value calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
PV(FCF at $\rho$ )	45,998.22	45,998.22	53,082.73	61,849.91	70,883.36
PV(TS at $\rho$ )	2,490.99	2,490.99	1,651.12	913.39	337.25
Total	48,489.22				

Capital cash flow and APV with the TS discounted at  $\rho$ , are the same.

Table 29 CFE, and market value of equity calculation with CFE at e

	Year 0	Year 1	Year 2	Year 3	Year 4
CFE=FCF+TS-CFD		4,596.28	5,783.79	8,843.89	91,964.55
PV(CFE at e)	32,316.43	32,316.43	42,631.36	54,703.63	67,192.55

In table the resulting e and equity market values are shown. It has to be remembered that there exists circularity and this is the result of iterations done by the spreadsheet.

Now, the MM WACC approach is presented.

Table A30 Basic information for MM WACC calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax rate		40.00%	40.00%	40.00%	40.00%
Cost of debt before taxes		28.55%	28.55%	28.55%	28.55%
Given nominal $\rho$ year 1 and assuming constant risk		40.15%	38.90%	37.65%	36.40%
Real $\rho$ (deflated)		25.13%	25.13%	25.13%	25.13%
Discount factor at $\rho$		0.7135	0.5137	0.3732	0.2736
FCF		11,383.78	11,881.29	14,251.39	96,682.05
Debt (balance)	16,110.00	12,082.50	8,055.00	4,027.50	
Initial Equity contribution (book value)	24,000.00				
Initial investment (fixed assets plus cash)	40,110.00				
Interest payments		4,600.00	3,450.00	2,300.00	1,150.00
Debt payment		4,027.50	4,027.50	4,027.50	4,027.50
Cash flow to debt before taxes		8,627.50	7,477.50	6,327.50	5,177.50
Tax savings (TS = TdD)		1,840.00	1,380.00	920.00	460.00

This information is common for the three methods, except for the tax savings TS. With the TV WACC approach, TS are calculated in an explicit way. This is, we determine how much TS is earned according to the financial situation of the firm. In this case we calculate the TS straightforward with the formula TdD.

Table A31 MM WACC and market value calculations Year 0  $\rho$  known

	Year 0	Year 1	Year 2	Year 3	Year 4
WACC calculations					
Debt					
Weight for Debt D%		33.41%	22.17%	12.87%	5.66%
After tax cost of debt		22.62%	22.62%	22.62%	22.62%
Contribution of debt to WACC		7.56%	5.02%	2.91%	1.28%
Equity					
Weight for Equity E%		66.59%	77.83%	87.13%	94.34%
$e_t = \rho_t + (1-T)(\rho_t - d)D_{t-1}/E_{t-1}$		43.64%	40.67%	38.45%	36.68%
Contribution of equity to WACC		29.06%	31.65%	33.51%	34.60%
WACC after taxes		36.6%	36.7%	36.4%	35.9%
Market Value at t @ WACC	48,225.41	54,501.59	62,604.27	71,151.08	

In table the resulting WACC and market values are shown. The HP WACC is calculated based on the contribution of each: the debt and the equity. It has to be remembered that there exists circularity and this is the result of iterations done by the spreadsheet.

Table A32 APV and market value calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
PV(FCF at $\rho$ )	45,998.2	45,998.2	53,082.7	61,849.9	70,883.4
PV(TS at d)	1,431.3	1,431.3	1,073.5	715.7	357.8
Total	47,429.5				

In this case, the CCF does not have any meaning. The FCF is discounted at  $\rho$  and the TS is discounted at d.

Table 33 CFE, and market value of equity calculation with CFE at e

	Year 0	Year 1	Year 2	Year 3	Year 4
CFE=FCF+TS-CFD		4,596.28	5,783.79	8,843.89	91,964.55
PV(CFE at e)		33,275.72	43,201.04	54,985.66	67,285.40

In table the resulting e and equity market values are shown. It has to be remembered that there exists circularity and this is the result of iterations done by the spreadsheet.