

A New WACC with Losses Carried Forward for Firm Valuation

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Abstract

Finance teachers teach that the present value of the free cash flow at the weighted average cost of capital less liabilities should be identical to the cash flow to equity discounted at the cost of equity capital, e . But usually, even though the cost of capital and the cost of equity are changing in time, the two cash flows are discounted at constant rates, and, at best, calculated with debt equity ratios based on book values.

In this paper the relationship between firm values calculated through the two cash flows are examined. Several examples and approaches are presented. It is shown that when market values are used to calculate the cost of capital and e , results are consistent. An adjusted version of the textbook formula for the cost of capital is presented. It takes into account the actual tax savings through recognizing the losses carried forward.

Keywords

Firm valuation, NPV, Free Cash Flow, FCF, Cash Flow to Equity, CFE, Cash Flow to Debt, CFD, Discounted Cash Flow, DFC.

JEL Classification: D92, E22, G12, G31, M40, M41, M46

A New WACC with Losses Carried Forward for Firm Valuation

Scholars and authors present in a heroic and elegant way, sometimes out of reality, the economic and financial models. Often, this approach might produce a sense of dissatisfaction among the readers (specially among skeptical students) to the point that they conclude that all that is taught is just rubbish. The theories are elegant and even beautiful, but they are not applicable, they say.

In a M & M world, the equity value is the present value of the Free Cash Flow FCF at the Weighted Average Cost of Capital WACC minus debt and it should be identical to the present value of the CFE (cash flow to equity) discounted at the cost of equity capital, e . This is correct. But usually, even though the WACC and the cost of equity are changing in time, the FCF and CFE are discounted at constant rates, and, in the best of cases, calculated with book value debt equity ratios.

In this paper the relationship between firm value calculated through the FCF and the CFE is examined. Several examples and approaches to the firm value calculations are presented. It is shown that when market values are used to calculate WACC and e , there are consistent results. The rate of discount to use for the tax savings is discussed. Also, an adjusted version of the textbook formula for WACC is presented. An Adjusted WACC that takes into account the actual tax savings through recognizing the losses carried forward LCF.

Many teachers might present the idea in an elegant way, for instance: "If you discount the CFE at an appropriate rate you will get the same results as if you discount the FCF at WACC and subtract the market value of liabilities". (Usually the market value of liabilities is what accountants register in the Balance Sheet). What is the appropriate rate? It is never said. Are teachers contributing thus, to the lack of respectability of the practitioners with respect to the academy?

Most finance textbooks (See Benninga and Sarig, 1997, Brealey, Myers and Marcus, 1996, Copeland, Koller and Murrin, 1994, Damodaran, 1996, Gallagher and Andrew, 2000, Van Horne, 1998, Weston and Copeland, 1992) present the Weighted Average Cost of Capital WACC calculation as:

$$\text{WACC} = d(1-T)D\% + eE\% \quad (1)$$

Where d is the cost of debt before taxes, T is the tax rate, $D\%$ is the percentage of debt on total value, e is the cost of equity and $E\%$ is the percentage of equity on total value. All of them are precise but not with enough emphasis on the fact that the values to calculate $D\%$ y $E\%$ are market values. Although they devote special space and thought to calculate d and e , little effort is made to show the correct calculation of market values. This means that there are several points that are not sufficiently dealt with:

1. Market values have to be calculated period by period and they are the present value at WACC of the future free cash flows.
2. The values to calculate $D\%$ and $E\%$ in the WACC are located at the beginning of period t . From here on, the right notation will be used.
3. $d(1-T)$ implies that the tax payments coincides in time with the interest payments. (Some firms could present this payment behavior, but it is not the rule. Only those that are subject to tax withheld from their customers pay taxes as soon as they pay interest charges).
4. The calculations for the cash flows usually depart from accounting figures such as, EBIT and this does not guarantee a proper treatment of tax savings.
5. Because of 1., 2. and the existence of changing macroeconomic environment, (say, inflation rates) WACC changes from period to period.
6. That there exists a circularity when calculating WACC. In order to know the firm value it is necessary the WACC, but to calculate WACC, the firm value and the financing profile are needed.

The purpose of this note is to clear up these ideas and emphasize in some issues that usually are looked over.

Some background in valuation

There are two broad classes of valuation methods: the accounting and the return or profitability methods. Accounting methods include book value, adjusted net assets value method, replacement value, and liquidation value. The return methods take into account the capacity of the firm to create value. They include stock market value, valuation by multiples and the discounted cash flow.

Regarding the different the finance metrics, Gravert, and Ramos, (1999), say:

"... The variety of finance metrics that has been increasing the traditional tools used to evaluate the firm performance and valuation, makes necessary to study them from the point of view of both, their relevance, and the incremental value of the information that they provide (Diaz, Riadi and Vidal, 1998). There are many discussions on development regarding the finance metrics (Dillon & Owners, 1997; Kaplan & Lowes, 1993). Among them, the following have a distinctive position: 1) The Discounted Cash Flow (DCF) published by Lek/Alkar and Alfred Rappaport (author of the Shareholder Value Analysis (SVA); 2) The Cash Flow Return on Investment - (CFROI) created by the Boston Consulting Group; 3) the Return on Invested Capital (ROIC), developed by McKinsey & Co., and the Economic Value Added (EVA) developed by Stern Stewart & Co. On the side of accounting metrics firstly developed by Du Pont in 1919, which makes identical the concept of Return on Investments (ROI) and Return on Assets (ROA), the interest is focused in the concept of accountability at different levels within an organization (Blymenthal, 1998). From all these metrics, certainly the more appealing is EVA (Myers, 1997).

The number of discrepancies between accounting measures of return and the true return are sufficiently known and widely discussed in the economic and finance literature. Harcour in 1965; Salomon and Laya in 1967; Livingston and Solomon in 1970; Fisher and McGowan in 1983 and Fisher in 1984 are conclusive regarding the differences existing between the measurement through accounting return rate, and the real rate of return of enterprises. The impact that inflation provokes in this discrepancy was cited by Salomon and Laya (1967); Kay (1976); Van Breda (1981); Kay and Mayer (1986), and De Villiers (1989) to demonstrate that inflation multiply the effects of the discrepancy between both methods (De Villiers, 1997, p. 286-287)..."

For an extensive bibliography see also, Gravert, and Ramos, (1999).

Of those methods, the most accepted one is the discounted cash flow method. This approach has two procedures: the sequential valuation procedure and the direct or discounted cash flow to equity procedure. The sequential valuation procedure consists of discounting the FCF at the WACC and the total or aggregate value of the firm is obtained. This includes debt and equity. From this total value, the values associated to the securities holders (different from stockholders) are subtracted to obtain the equity or firm value. The direct method discounts the CFE at the cost of equity and this result is the equity firm value.

There are two ways to get the FCF: Directly from the Profit and Losses statement P&L and from the Cash Budget.

From the P&L the method starts from the Earnings before Interest EBIT, adds depreciation and amortization charges, and subtracts taxes on EBIT and changes in non-cash working capital.

This method is widely used and is found in any financial textbook. However, it might be the source of potential errors. As it subtracts the tax on EBIT directly, it may be possible that earnings before Taxes EBT is zero or negative and then taxes are not paid; hence, the tax payment and the correspondent tax savings might never occur.

A simple example will illustrate this idea. Assume that EBIT is 100 and the tax rate is 40%. If taxes are calculated as usual, directly from EBIT, they will amount to 40 and if interest charges are 150 the tax saving will be accounted as 60. However, if interest charges are 150 taxes will be zero, but there exists a tax savings of only 40.

Table 1. Tax savings calculations

	Levered	Unlevered
EBIT	100	100
Interest charges	150	0
EBT	-50	100
Tax	0	40
Net profit	-50	60

On the other hand, it is the indirect calculation of FCF. Financial statements (Balance sheet, P&L and Cash Budget CB) are projected and from the CB, the FCF is derived. The cash budget procedure, as presented here, is not found in most textbooks.

It is important to know the performance of the firm or project in terms of liquidity. A careful examination of the cash budget CB will allow the decision maker to choose a financing alternative or, on the other hand, a good investment of cash surplus decision.

The elements of the cash budget CB are the inflows and the outflows of cash. The difference between these two figures result in the cash balance of the period and from it, the cumulative cash balance can be calculated. This cash balance is the clue to decide if the firm should borrow or invest funds.

Table 2. Typical items included in the Cash Budget CB

Inflows	Outflows
Accounts receivables recovery	Accounts payable payments
Loans received	Salaries and fringe benefits
Equity invested	Interest charges
Sale of assets	Principal payments
Return on short or long term investment	Rent
Short term investment recovery	Overhead expenses
Customers' in advance payments	Promotion and publicity
Repayments of cash lent	Asset acquisition
	Social Security payments
	Earnings distributed or dividends paid
	Taxes
	Short term investment of cash surpluses

In Velez-Pareja (1999) the procedure for constructing the cash flows is presented. There, the FCF and the CFE are defined. The approach is to depart from the Cash Budget or Working Cash Statement. The most important tool for financial management is the Cash Budget that discloses the cash situation. The procedure to calculate the cash flows is

The FCF “[...], for year 1 to n:

Net cash gain (loss) after financing and reinvestment from the CB (NCG)
Minus investment from stockholders (IS)
Minus proceeds from loans received (LR)
Plus principal payments (PP)
Plus interest, rent and lease charges paid (I)
Minus tax shield for interest, rent and lease payments (TSI)
Plus dividends or earnings paid (D)
Minus investment in the project (IP)
Equal free cash flow

At period n, the present value of future FCF beyond the period n, has to be added”.

The CFE “[...] for periods 1 to n-1

Net cash gain (loss) after financing and reinvestment from CB (NCG)
Minus investment from stockholders (IS)
Plus dividends or earnings paid (D)
For period 0, the cash flow is the total investment from stockholders.
For period n:
Cumulative cash balance at end of year after financing and reinvestment (CCB)
Minus investment from stockholders (IS)
Plus net terminal value (NSV)
Plus dividends or earnings paid (D)”

However, if the FCF and the CFD are known, then the CFE might be calculated simply as $CFE = FCF - CFD$.

Another reason to endorse the CB method to calculate the FCF is the usefulness of the CB as a financial management tool.

Benninga and Sarig (1997) have some arguments to support that the firm valuation should be done from the FCF -the sequential procedure- and not from CFE -the direct procedure. Benninga and Sarig (1997, p. 411) argue, to support the use the FCF as a means to determine the firm value, that using the sequential procedure, the “valuation of equity is *independent of the dividend stream*” based on the Modigliani- Miller (MM) (1961) assumptions. On the other hand, Copeland et al (1995, p. 149) support the sequential procedure (they call it the entity model) and say with respect to the direct procedure:

“[...] this approach is not as useful as the entity model. Discounting equity cash flow provides less information about the sources of value creation and is not as useful for identifying value creation opportunities. Furthermore, it requires

careful adjustments to ensure that changes in projected financing do not incorrectly affect the company value. [...] Increasing the dividend payout ratio requires more use of debt. More debt means riskier equity and a higher discount rate for the equity.”

This might be true if the model does not keep track all these changes. The valuation model has to adjust interest rates (debt and equity cost) with leverage.

Damodaran (1996, p. 10) considers that “while the two approaches use different definitions of cashflows and discount rates, they will yield consistent estimates of value as long as the same set of assumptions is used for both. The key error to avoid is mismatching cashflows and discount rates [...]”

Statements such as these will be examined in this paper.

While academics are struggling on how to calculate correctly the WACC, practitioners, on the other hand, do not devote too much effort in that task. They say it is not worth it. In Annex 2, a transcript of the opinions of two chief financial officers is presented. One from Europe, posted at Japan and another one from Latin America, posted at Philippines. Their testimonies give support to what is said in this paper.

The Modigliani-Miller Proposal

The basic idea is that the firm value does not depend on how the stakeholders finance it. This is, the stockholders (equity) and creditors (liabilities to banks, bondholders, etc.) The reader should examine this idea in an intuitive manner and she will find it reasonable. Because of this idea, Franco Modigliani and Merton Miller (MM from here on) were awarded the Nobel Prize in Economics. They proposed that with perfect market conditions, (perfect and complete information, no taxes, etc.) the capital structure does not affect the value of the firm because the equity holder can borrow and lend and thus determine the optimal amount of leverage. The capital structure of the firm is the combination of debt and equity in it.

That is, for any year n , V_n^L the value of the levered firm is equal to V_n^{UL} the value of the unlevered firm.

$$V_n^L = V_n^{UL} \quad (2)$$

And in turn, the value of the levered firm is equal to V_n^{Equity} the value of the equity plus V_n^{Debt} the value of the debt.

$$V_n^L = V_n^{\text{Equity}} + V_n^{\text{Debt}} = E + D \quad (3)^2$$

What does it imply regarding the discount rate? Simple. If the firm has a given cash flow, the present value of it (the firm total value) does not change if the capital structure changes. If this is true, it implies that the discount rate will remain constant no matter how the capital structure changes. This constant discount rate is ρ the cost of capital *before* taxes. This situation happens when no taxes exist. To maintain the equality of the unlevered and levered firms, the return to the equity holder (levered) must change with the amount of leverage (assuming that the cost of debt is constant).

One of the main market imperfections is tax. When corporate taxes exist (and no personal taxes), the situation posited by MM is different. They proposed that when taxes exist the total value of the firm does change. This occurs because no matter how well managed is the firm; if it pays taxes, there exists what economists call an externality. When the firm deducts any expense, the government pays a subsidy for the expense. It is reflected in less tax. In particular, this is true for interest payments. The value of the subsidy (the tax saving) is TdD , where the variables have been defined above. Hence the value of the firm is increased by the present value of the tax savings or tax shield.

$$V_n^L = V_n^{UL} + V_n^{\text{TS}} \quad (4)$$

When a firm has debt there exists some other contingent or hidden costs associated to the fact to the possibility that the firm goes to bankruptcy. Then, there are some expected costs that could reduce the value of the firm. The existence of these costs deters the firm to take leverage up to 100%. One of the key issues is the appropriate discount rate for the tax shield. In this note, it is asserted that

the correct discount rate for the tax shield is ρ , the return to unlevered equity, and the choice of ρ is appropriate whether the percentage of debt is constant or varying over the life of the project.

Now the effects of taxes on the WACC will be studied. When calculating WACC two situations can be found: with or without taxes. In the first case, as said above, the WACC is constant and equal to ρ , no matter how the firm value be split between creditors and stockholders. (The assumption is that inflation is kept constant, otherwise, the ρ will change accordingly). When inflation is not constant, ρ changes, but due to the inflationary component and not due to the capital structure. In this situation, WACC is the cost of the assets or the cost of the firm, ρ and at the same time is the cost of equity when unlevered. This means,

$$\rho_t = dD_{t-1}\% + e_t E_{t-1}\% \quad (5)$$

This ρ is defined as the return to unlevered equity. The WACC is defined as the weighted average cost of debt and the cost of levered equity. In a MM world ρ is equal to WACC without taxes.

If it is true that the cost ρ is constant, e , the cost of equity changes according to the leverage. Here for simplicity it is assumed that the ρ is constant, but this assumption is not necessary. If the ρ is changing then in each period, the WACC will change as well, not only for the eventual change in the financing profile, but for the change in ρ . In any case, e has to change in order to keep ρ constant

The cost of equity, e is

$$e_t = (\rho_t - d D\%_{t-1}) E\%_{t-1} / \rho_t + (\rho_t - d) D\%_{t-1} / E\%_{t-1} \quad (6)$$

When taxes exist, the WACC calculation will change taking into account the tax savings. Hence, WACC after taxes will be calculated as

$$WACC_t = d_t(1-T) D\%_{t-1} + e_t E\%_{t-1} \quad (7)$$

The values for $D\%$ and $E\%$ have to be calculated on the total value of the firm for the beginning of each period.

If the Capital Asset Pricing Model (CAPM) is used, it can be demonstrated that there is a relationship between the betas of the components (debt and equity) in such a way that

$$\beta_{t \text{ firm}} = \beta_{t \text{ debt}} D_{t-1}\% + \beta_{t \text{ stock}} E_{t-1}\% \quad (8)$$

If $\beta_{t \text{ stock}}$, $\beta_{t \text{ debt}}$, $D_{t-1}\%$ and $E_{t-1}\%$ are known, then ρ can be calculated as

$$\rho = R_f + \beta_{t \text{ firm}} (R_m - R_f) \quad (9)$$

Where R_f is the risk free rate and $(R_m - R_f)$ is the market risk premium. Known ρ , e can be calculated for any period.

The WACC deducted from the Cash Flows

First, it is shown that the relationship $PV(\text{FCF}) = PV(\text{CFE}) + PV(\text{CFD})$ holds. Then based on the equality, the expression for the WACC is found. In fact, if it is assumed that the correct discount rate for the tax shield is the cost of debt d , then the traditional formula for the WACC, namely,

$$\text{WACC} = d(1-T) D\% + eE\% \quad (10)$$

does not hold for any project with finite cash flows.

When it is assumed that the correct discount rate for the tax shield is the cost of debt d , as Miles and Ezzell (ME, 1980) do³, the discount rate for the FCF is $\rho^* = \rho - TdD\%(1 + \rho)/(1 + d)$. This expression is not consistent with the traditional WACC, as will be seen below.

The usual position regarding the correct discount rate for the tax shield is:

1. If the amount of debt is known, then the correct discount rate for the tax shield is d , the cost of debt before taxes.
2. If the amount of debt is a fixed proportion of the total value, then the discount rate is ρ
3. If the amount of debt is unknown, the correct discount rate for the tax shield is ρ

The Rate of Discount for the Tax Savings

Miles and Ezzel (ME, 1980) proposed that the discount rate for an uneven levered FCF for $N > 1$ is

$$\rho^* = \rho - TdD\%(1 + \rho)/(1 + \psi) \quad (11)$$

Where ψ is the rate for discounting the tax shield and $D\%$ is constant. This expression should be considered as the basic one for WACC. On the other hand, they pose that the traditional textbook formula for WACC after taxes is

$$\text{WACC} = eE\% + d(1-T)D\% \quad (12)$$

They assume that as debt is known for the first period and unknown for the rest, then the correct discount rate for tax savings is ρ for periods 2 to N and d for period 1. However, the reason for using ρ for discounting the TS is that TS depends on the returns of the firm. Depending on them there will be taxes to pay and hence, a tax saving on interest charges. With this simple argument, the discount rate for period 1 should be set at ρ . However, let us assume that ψ is unknown, but it is necessary to make it consistent with WACC definition.

If $\text{WACC} = \rho^*$, then

$$\rho - TdD\%(1 + \rho)/(1 + \psi) = eE\% + d(1-T)D\% \quad (13a)$$

$$\rho(1 + \psi) - TdD\%(1 + \rho) = (1 + \psi)(eE\% + d(1-T)D\%) \quad (13b)$$

$$\rho(1 + \psi) - (1 + \psi)(eE\% + d(1-T)D\%) = TdD\%(1 + \rho) \quad (13c)$$

$$(1 + \psi)(\rho - (eE\% + d(1-T)D\%)) = TdD\%(1 + \rho) \quad (13d)$$

$$(1 + \psi)(\rho - eE\% - dD\% + dTD\%) = TdD\%(1 + \rho) \quad (13e)$$

$$(1 + \psi)(\rho - \rho + dTD\%) = TdD\%(1 + \rho) \quad (13f)$$

$$\psi = \rho \quad (13g)$$

The discount rate for the tax savings is $\psi = \rho$

Myers (1974) and most finance textbooks stipulate that the discount rate for the tax savings should be d , the cost of debt before taxes. However, an inconsistency arises when discounting the FCF at WACC and calculating the Adjusted Present Value using d as the discount rate for the tax savings. It has been shown by Tham (1999) that for the $n=1$ and perpetuity case, the proper discount rate is not d , but ρ .

Tham and Velez-Pareja (2000) using no-arbitrage arguments have shown that this holds for any n . As seen above, this also can be deduced from Miles and Ezzel (1980). The total firm value is

$$\text{PV of unlevered cash flow at } \rho + \text{PV of tax savings at } \rho \quad (14a)$$

The only thing to note is that the tax shield is discounted at ρ and not at d , the cost of debt.

This approach is the same as the Capital Cash Flow CCF, presented by Ruback, 2000. CCF is just FCF plus TS. Then,

$$\text{Total value} = \text{PV of CCF at } \rho \quad (14b)$$

Again, the equity value can be deducted from the calculated total value of the firm, subtracting the present value (usually the book value) of the debt.

The basic intuition is that the risk of the tax shield is the same as the free cash flow.

An Adjusted WACC

The ME results define the correct discount rate for the FCF as

$$\rho^* = \rho - \frac{TdD}{1+\rho} \quad (15)$$

They use ψ as d . However, as was mentioned above, this is not consistent with the textbook WACC. When $\psi = \rho$, (33) is the same expression presented by Harris and Pringle (1985). However, it is possible to think of an Adjusted WACC, consistent with the traditional WACC. In fact, it is the same.

The traditional formula is:

$$\text{WACC} = d(1 - T) D\% + eE\% = dD\% - TdD\% + eE\% \quad (16)$$

What is the meaning of the second term on the right? It is simply the subtraction of the tax savings from the interest deduction. The idea is then to replace $TdD\%$ for its actual value.

$$\text{WACC} = eE\% + d(1-T)D\% \quad (17a)$$

$$\text{WACC} = eE\% + dD\% - TdD\% \quad (17b)$$

$$\text{WACC} = eE\% + dD\% - TdD/(\text{Total levered value TLV}) \quad (17c)$$

$$\text{WACC} = eE\% + dD\% - TS/TLV \quad (17d)$$

$$\text{Adjusted WACC} = \text{TV WACC} = \rho - TS/TLV \quad (17e)$$

In the calculation of the cost of capital, one key issue is whether the tax shield is included in the free cash flow or not. It can be included or not. Most corporate finance textbooks prefer to exclude it. But the great thing is that it does not matter.

The Tham Velez (TV) WACC proposed here might be a bit difficult to calculate because it is necessary to compare the tax savings from the tax shields with and without losses carried forward. The reason it is a very strange formula is because it takes into account the fact that losses are carried forward. This is a very important issue because most textbook formulas assume that the tax shield is always used in each year. This TV WACC might be difficult to teach and understand. In other words, in the same way as the after tax WACC has to consider when the tax shield is earned and not blindly use the $(1-T)$ factor, in the same way, it is necessary to take into account if the tax shield is realized or not. This formula shows in principle that the traditional formula for the WACC can be adjusted to take into account the losses carried forward.

The value of TS will depend on EBIT and if losses are carried forward or not.

These are some possible cases:

First, taxes are paid the same year as accrued.

Case 1. $\text{EBIT}_t \geq dD_t = I_t$ $TS_t = T I_t$

Case 2. $EBIT_t = 0$ $TS_t = 0$

Case 3. $EBIT_t < I_t$

Sub case a) No LCF $TS_t = T EBIT_t$

Sub case b) LCF and assuming LCF are recovered totally the following year.

$$TS_t = T(I_t + \text{Losses}_{t-1}) = T(I_t + L_{t-1})$$

Hence, the expression for WACC should be

$$WACC_t = \rho - T(I_t + L_{t-1}) / (\text{Total Levered Value}_{t-1}) = \rho - T(I_t + L_{t-1}) / (TV_{t-1}) \quad (18a)$$

if the next year after losses . $EBIT_t \geq dD_t + L_{t-1}$

$$WACC_t = \rho - TEBIT_t / TV_{t-1} \quad (18b)$$

if the year of losses . $EBIT_t < I_t$ and no LCF

$$WACC_t = \rho - TI_t / TV_{t-1} \quad (18c)$$

when no losses ($EBIT_t \geq dD_t = I_t$)

Second, taxes are paid the next year as accrued.

Case 1. $EBIT_t \geq dD_t = I_t$ $TS_{t+1} = T I_t$

Case 2. $EBIT_t = 0$ $TS_{t+1} = 0$

Case 3. $EBIT_t < I_t$

Sub case a) No LCF $TS_{t+1} = T EBIT_t$

Sub case b) LCF and assuming LCF are recovered totally the following year.

$$TS_{t+1} = T(I_t + \text{Losses}_{t-1}) = T(I_t + L_{t-1})$$

Hence, the expression for WACC should be

$$WACC_t = \rho - T(I_{t-1} + L_{t-2}) / (\text{Total Levered Value}_{t-1}) = \rho - T(I_{t-1} + L_{t-2}) / (TV_{t-1}) \quad (18d)$$

if the next year after losses $EBIT_t \geq dD_t + L_{t-1}$

$$WACC_t = \rho - TEBIT_{t-1} / TV_{t-1} \quad (18e)$$

if the year of losses . $EBIT_t < I_t$ and no LCF

$$WACC_t = \rho - TI_{t-1}/TV_{t-1} \quad (18f)$$

when no losses ($EBIT_t \geq dD_t = I_t$)

The Adjusted WACC is closest in spirit to the textbook formula because it is applied to the free cash flow net of tax, excluding the value of the tax shield. The benefit of the tax shield is taken into account by lowering (adjusting) the WACC.

The strength of Adjusted WACC is to take into account the TS when and in the amount they occur. This is, it takes into account the LCF.

The total value of the firm can be calculated from the FCF discounted at WACC or better at Adjusted WACC, to take into account any losses carried forward LCF. This means that the model to calculate the FCF has to take into account the LCF. Also, this means that the simple approach to deduct the FCF from EBIT, NOPAT or net profit has to be used carefully and properly adjusted.

The equity value can be deducted from the calculated total value of the firm, subtracting the present value (usually the book value) of the debt.

The other weakness of the traditional WACC is that it does not take into account losses carried forward. Unfortunately, with losses carried forward, the traditional formula for the WACC is unable to handle it, even if it is assumed that the debt equity ratio is constant and thus, the traditional WACC would be constant. In practice, losses carried forward are a critical issue.

The Cost of Equity, e

The equity value can be calculated from the CFE discounted at the *periodically changing* cost of equity e , as a function of a *periodically changing* debt to equity ratio.

Again note that from (6) the formula for e is

$$e_t = (\rho_t - d D\%_{t-1})/E\%_{t-1} = \rho_t + (\rho_t - d)D\%_{t-1}/E\%_{t-1} \quad (19)$$

This is a key point. The traditional formula in the textbook is:

$$e = \rho_t + (1-T)(\rho_t - d)D\%_{t-1}/E\%_{t-1} \quad (20a)$$

Note that in the formula for the calculation of e (6), the return to levered equity, there is no adjustment factor of $(1-T)$. The reason for this is as follows. When ρ is used as the discount factor for the tax shield, the factor $(1-T)$ drops out in the formula for e . This can be verified with algebra.

Hence, the formula that should be used to produce consistent results is:

$$\text{WACC} = \rho - \text{TS/TV} \quad (20b)$$

Practitioners and even some scholars seem to be perfectly happy with using a constant e even though they know that the debt-equity ratio is changing. This seems to be inconsistent and they will recognize it as inconsistent if it is pointed out, but usually they ignore it.

The Million-Dollar Question: How to find r ?

Or equivalently, how e is found? If it is possible to find ρ , then e is known. And if e is known, then, ρ known. When these ideas are presented, people have no problem with e . That is, based on their experience as investors, they are willing to give a value for e , even though they may not adjust e as the debt-equity ratio changes. But the point is that **if e is known, then r is also known.**

Calculations for e and r

The secret is to calculate e or ρ . If e is known for a given period, the initial period, for instance, ρ can be calculated. On the contrary, if ρ is known e can be calculated. For this reason several options to calculate e and ρ are presented.

In order to calculate e , there are several alternatives:

1. With the Capital Asset Pricing Model, CAPM. This is the case of a firm that is traded at the stock exchange, it is traded on a regularly basis and it is believed that the CAPM works well.
2. With the Capital Asset Pricing Model, CAPM adjusting the betas. This is the case for a firm that is not registered at the stock exchange or if registered, is not frequently traded and it is believed that the model works well. It is necessary to pick a stock similar to the one under

study, (from the same industrial sector, about the same size and about the same leverage). This is called the proxy firm.

Example:

The beta adjustment is made with

$$\mathbf{b}_{nt} = \mathbf{b}_{proxy} \frac{\left[1 + \frac{D_{nt}}{E_{nt}}(1-T) \right]}{\left[1 + \frac{D_{proxy}}{E_{proxy}}(1-T) \right]} \quad (21)$$

Where, β_{nt} is the beta for the stock not traded at the stock exchange; D_{nt} is the market value of debt, E_{nt} is the equity for the stock not registered in the stock exchange; D_{proxy} is the market value of debt for the proxy firm, E_{proxy} is the market value of equity for the proxy firm.

For instance, if you have a stock traded at the stock exchange and the beta is $\beta_{proxy} = 1.3$, a debt $D_{proxy} = 80$, $E_{proxy} = 100$, and it is desired to estimate the beta for a stock not traded in the stock exchange. This non-traded stock has a debt $D_{nt} = 70$ and an equity of $E_{nt} = 145$ and a tax rate of 35%, then beta for the non-traded stock can be adjusted as

$$\mathbf{b}_{nt} = \mathbf{b}_{proxy} \frac{\left[1 + \frac{D_{nt}}{E_{nt}}(1-T) \right]}{\left[1 + \frac{D_{proxy}}{E_{proxy}}(1-T) \right]} = 1.3 \frac{\left[1 + \frac{70}{145}(1-35\%) \right]}{\left[1 + \frac{80}{100}(1-35\%) \right]} = 1.12$$

3. Subjectively and assisted by a methodology such as the one presented by Cotner y Fletcher, 2000 and applied to the owner of the firm. With this approach, the owner given a leverage level estimates the perceived risk. This risk premium is added to the risk free rate and the result would be an estimate for e.
4. Subjectively as in 3., but direct. This is, asking the owner, for a given value level of debt and a given cost of debt, what is the required return to equity?

5. An estimate based on book value (given that these values are adjusted either by inflation adjustments or asset revaluation, so the book value is a good proxy to the market value).

Here is an example. Assume a privately held firm. The tax rate is 35%

Table 4. Financial information of hypothetical firm

Year	Adjusted book value for equity E	Dividends paid D	Return $R_t = ((D_t + E_t)/E_{t-1}) - 1$
1990	\$1,159	\$63	
1991	\$1,341	\$72	21.92%
1992	\$2,095	\$79	62.12%
1993	\$1,979	\$91	-1.19%
1994	\$3,481	\$104	81.15%
1995	\$4,046	\$126	19.85%
1996	\$3,456	\$176	-10.23%
1997	\$3,732	\$201	13.80%
1998	\$4,712	\$232	32.48%
1999	\$4,144	\$264	-6.45%
2000	\$5,950	\$270	50.10%

Table 5. Additional macroeconomic information

Year	Nominal risk free rate of interest ⁴ R_f	CPI	Inflation $i_f = (CPI_t/CPI_{t-1}) - 1$	Real interest rate $i_r = (1+R_f)/(1+i_f) - 1$	Return to equity $e_t = ((D_{t+1} + E_{t+1})/E_t) - 1$	Risk premium $i_0 = e_t - R_{ft} \times (1-T)$
1990	36.3%	166.94				
1991	30.6%	211.72	26.8%	3.0%	21.92%	2.0%
1992	28.9%	264.94	25.1%	3.0%	62.12%	43.3%
1993	26.3%	324.84	22.6%	3.0%	-1.19%	-18.3%
1994	26.3%	398.24	22.6%	3.0%	81.15%	64.1%
1995	15.8%	475.76	19.5%	-3.1%	19.85%	9.6%
1996	16.3%	578.71	21.6%	-4.4%	-10.23%	-20.8%
1997	21.2%	681.06	17.7%	3.0%	13.80%	0.0%
1998	51.7%	794.80	16.7%	30.0%	32.48%	-1.1%
1999	16.4%	898.12	13.0%	3.0%	-6.45%	-17.1%
2000	12.9%	984.34	9.6%	3.0%	50.10%	41.7%
2001			Expected 10%	Average 4.4%		Average 10.3%

Estimated after tax risk free rate for 2001:

$$R_{f2001} = ((1+i_{f\text{ est.}})(1+i_{\text{prom.}}) - 1) \times (1-T)$$

$$= ((1+10\%)(1+4.4\%) - 1) \times (1-0.35) = 9.61\%$$

Cost of equity $e = R_{f2001} + i_{0\text{ average}} = 9,61\% + 10,30\% = 20,0\%$

6. Calculate the market risk premium as the average of $R_m - R_f$, where R_m is the return of the market based upon the stock exchange index and R_f is the risk free rate (say, the return of treasury bills or similar). Then, subjectively, the owner could estimate if he prefers, in terms of risk, to stay in the actual business or to buy the stock exchange index basket. If the actual business is preferred, then one could say that the beta of the actual business is lower than 1, the market beta, and the risk perceived is lower than the market risk premium, $R_m - R_f$. This is an upper limit for the risk premium of the owner. This upper limit could be compared with zero risk premium, the risk free rate risk premium which is the lower limit for the risk perceived by the equity owner.

If the owner prefers to buy the stock exchange index basket, it could be said that the actual business is riskier than the market. Then, the beta should be greater than 1 and the perceived risk for the actual business should be greater than $R_m - R_f$.

In the first case, the owner could be confronted with different combinations -from 0% to 100%- of the stock exchange index basket and the risk free investment and the actual business. After several trials, the owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk, or simply, the market risk premium ($R_m - R_f$) times the proportion of the stock exchange index basket accepted. In fact what has been found is the beta for the actual business.

In the second case one must choose the highest beta found in the stock exchange index basket. (The stock exchange or any governmental control office usually calculates these betas. In Colombia the betas for each stock are calculated by the *Superintendencia de Valores*, similar to the Stock Exchange Commission, SEC). This beta should be used to multiply the

market risk premium $R_m - R_f$, and the result would be an estimate of the risk premium for the riskiest stock in the index. This might be an upper limit for the risk perceived by the owner. In case this risk is lower than the perceived risk by the owner, it might be considered as the lower limit. In case that the riskier stock is considered riskier than the actual business, then the lower limit is the market risk premium, $R_m - R_f$. In this second case, the owner could be confronted with different combinations -from 0% to 100%- of the stock exchange index basket and the riskiest stock and the actual business. After several trials, the owner eventually will find the indifference combination of risk free and the stock exchange index basket. The perceived risk could be calculated as a weighted risk. That is, the market risk premium ($R_m - R_f$) times the proportion of the stock exchange index basket accepted plus the risk premium for the riskiest stock in the index (its beta times the market risk premium, $R_m - R_f$) times the proportion accepted for that stock.

In both cases the result might be an estimation of the risk premium for the actual business. This risk premium could be added to the risk free rate (using Fisher Theorem), and this might be a rough estimate of e .

If e , $D\%$ and $E\%$ are known, then ρ is calculated with (5). Here circularity is found, but it is possible to solve it with a spreadsheet.⁵

Another option is to calculate ρ directly. One of the following alternatives could be used:

1. According to MM, the WACC before taxes (ρ) is constant and independent from the capital structure of the firm. Then one could ask the owner for an estimate on how much she is willing to earn assuming no debt. A hint for this value of e could be found looking how much she could earn in a risk free security when bought in the “secondary” market. On top of this, a risk premium, subjectively calculated must be included, as in 3. above.

2. Another way to estimate ρ is assessing subjectively *the risk for the firm* and this risk could be used to calculate ρ using the Fisher Theorem with the risk free rate. (Cotner and Fletcher, 2000 present a methodology to calculate the risk of a firm not publicly held⁶). This methodology might be applied to the managers and other executives of the firm. This would give the risk premium for the firm. As this risk component would be added to the risk free rate, the result is ρ calculated in a subjective manner. A hint that could help in the process is to establish minimum or maximum levels for this ρ . The minimum could be the cost of debt before taxes. The maximum could be the cost of opportunity of the owners, if it is perceptible (that is, if it has been “told” by them or if, by observation, it is known observing how they invest (other investments made by them).

This ρ is in accordance to the actual level of debt. It has to be remembered that ρ , is according to MM, constant and independent from the capital structure of the firm.

This ρ is named in other texts as K_A cost of the assets or the firm, (for instance, Ruback, 2000) or K_u cost of unlevered equity (for instance, Fernandez, 1999a y 1999b).

If ρ is calculated directly and it is necessary to estimate the WACC (or the e), circularities will be present. However, as will be shown below, the total value of the firm can be calculated with ρ no circularities will be present and here is no need for calculating the leverage ratio for every period

An Example for Calculating TV WACC and the Total Firm and Equity Value from FCF and CFE

For a better understanding of these ideas, an example is presented.

The information about cost of debt and unlevered equity ρ tax rate, the investment, free cash flows, debt balances and initial equity is

Table 6 Basic information for TV WACC calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax rate		40.00%	40.00%	40.00%	40.00%
Cost of debt before taxes		28.55%	28.55%	28.55%	28.55%
Given nominal ρ year 1 and assuming constant risk		40.15%	38.90%	37.65%	36.40%
Real ρ (deflated)		25.13%	25.13%	25.13%	25.13%
Discount factor at ρ		0.7135	0.5137	0.3732	0.2736
FCF		11,383.78	11,881.29	14,251.39	96,682.05
Debt (balance)	16,110.00	12,082.50	8,055.00	4,027.50	
Initial Equity contribution (book value)	24,000.00				
Initial investment (fixed assets plus cash)	40,110.00				
Interest payments		4,600.00	3,450.00	2,300.00	1,150.00
Debt payment		4,027.50	4,027.50	4,027.50	4,027.50
Cash flow to debt before taxes		8,627.50	7,477.50	6,327.50	5,177.50
Tax savings (TS)		0	1,380.00	920.00	460.00
Earnings before taxes		(46.34)	3,748.76	8,631.11	13,988.38
Taxes		0	1,480.97	3,452.44	5,595.35
Net profit		(46.34)	2,267.80	5,178.67	8,393.03

Observe that taxes paid at year 2 are a little less than 40% because the LCF from year 1. With the TV WACC approach, TS are calculated in an explicit way. This is, one determines how much TS is earned according to the financial situation of the firm. The TS is not calculated straightforward with the formula TdD . The exact calculations are $(3,748.76 - 46.34) \times 40\% = 1,480.97$.

The WACC calculations are made estimating the tax savings that are expected to occur and the market value of the firm.

Table 7 TV WACC and market value calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax savings (TS)		0	1,380.00	920.00	460.00
Market Total Value at t @ TV WACC	47,176.34	54,733.85	62,763.30	71,220.61	
TS/TV		0.00%	2.52%	1.47%	0.65%
WACC after taxes = $\rho - TS/Total\ value\ at\ t-1$		40.15%	36.38%	36.18%	35.75%

Observe that at year 1, TV WACC is equal to ρ , as expected, given that no tax savings were earned. Notice that WACC results in a lower value than ρ . WACC is after taxes.

Example: Firm value at end of year 3 is

$$96,682.05/(1+WACC_4) = 96,682.05/(1+35.75\%) = 71,220.61.$$

For the end of year 2 it will be

$$(71,220.61 + 14,251.39)/(1+WACC_3) = (71,220.61 + 14,251.39)/(1+36.18\%) = 62,220.61$$

and so on for the other years.

The reader has to realize that the values 35.75%, 36.18%, etc. are not given because they depend on the firm value that is going to be calculated with the WACC. In this case circularity is generated. This is solved allowing the spreadsheet to make enough iteration until it finds the final numbers. See Annex 1.

Calculating the firm value using r

Using the MM approach on the case with taxes the same result can be reached calculating the present value for the free cash flow assuming no debt and discount it at a ρ , or what is the same, at WACC before taxes and add up the present value of tax savings at the same rate of discount, ρ . Myers proposed this in 1974 and it is known as Adjusted Present Value APV. Myers and all the finance textbooks teach that the discount rate should be the cost of debt. However, the tax savings depend on the firm profits. Hence, the risk associated to the tax savings is the same as the risk of the cash flows of the firm rather than the value of the debt. Hence, the discount rate should be ρ . For this reason the

tax savings are also discounted at ρ . This way, the present value for the free cash flows discounted at WACC after taxes coincides with the present value of the free cash flow assuming no debt discounted at ρ and added with the present value of the tax savings discounted at the same ρ .

The use of ρ to discount the tax savings has been proposed by Tham, 1999, Tham, 2000 and Ruback, 2000. Tham proposes to add to the total value of the firm (the present value of the FCF at ρ), the present value of the tax savings discounted at ρ . Ruback presents the Capital Cash Flow and discount it at ρ . The CCF is simply the FCF plus the tax savings so,

$$\text{CCF} = \text{FCF} + \text{Tax savings} \quad (22)$$

$$\text{Total value} = \text{PV}(\text{FCF without debt at } \rho) + \text{PV}(\text{Tax savings at } \rho) \quad (23a)$$

$$\text{Total value} = \text{PV}(\text{CCF at } \rho) \quad (23b)$$

Table 8 APV and market value calculations

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax savings (TS)		0	1,380.00	920.00	460.00
FCF		11,383.78	11,881.29	14,251.39	96,682.05
PV(FCF at ρ)	45,998.22	45,998.22	53,082.73	61,849.91	70,883.36
PV(TS at ρ)	1,178.11	1,178.11	1,651.12	913.39	337.25
Total	47,176.34				

Table 9 Capital Cash Flow and market value calculations at ρ

	Year 0	Year 1	Year 2	Year 3	Year 4
Tax savings (TS)		0	1,380.00	920.00	460.00
FCF		11,383.78	11,881.29	14,251.39	96,682.05
Capital cash flow (CCF = FCF + TS)		11,383.78	13,261.29	15,171.39	97,142.05
Total value	47,176.34	54,733.85	62,763.30	71,220.61	

Capital cash flow and APV with the TS discounted at ρ , are the same.

The Firm Value from the CFE

From the point of view of firm valuation, its value is calculated with the present value of the free cash flow discounted at WACC minus the debt at 0. This value also can be reached with the equity cash flow (CFE) and it is equal to

$$CFE = FCF - CFD$$

$$CFE = FCF + TS - \text{Cash flow to debt before taxes CFD} \quad (24)$$

Table 10 CFE, e and market value of equity calculation with CFE at e

	Year 0	Year 1	Year 2	Year 3	Year 4
$e = \rho + (\rho - d)D/E$		46.16%	41.83%	38.99%	36.87%
$CFE = FCF + TS - CFD$		2,756.28	5,783.79	8,843.89	91,964.55
PV(CFE at e)		31,066.34	42,651.35	54,708.30	67,193.11
Value of equity PV(FCF) - Debt		31,066.34			

In this table the resulting e and equity market values are shown. The procedure to calculate e and equity market value is the same as the one mentioned for TV WACC. The values for e and for equity market value are not given. They result after the iterations in the spreadsheet to solve the circularity. It has to be remembered that there exists circularity and this is the result of iterations done by the spreadsheet.

Note that the cost of equity –e– is larger than ρ . And this what is expected, because ρ is the cost of the stockholder, as if there were no debt⁷. When there is debt –e calculation– necessarily e ends up being greater than ρ , because of leverage. With these values it is possible to calculate the firm value for each period.

If e is known, as it was said above, ρ is found with (5). Excel solves the circularity that is found and the same values result.

When the present value of CFE at e is calculated the same result is obtained. This is, $47,176.34 - 16,110 = 31,066.34$. This means that the right discount rate to discount the CFE is e , and its discounted value is consistent with the value calculated with the FCF.

In summary, from the example developed above,

Method	Total Value	Equity Value = Total Value - Debt = PV(CFE at e)
PV(FCF at WACC)	47,176.34	31,066.34
PV(FCF at ρ) + PV(Tax savings at ρ) = PV(FCF+TS at ρ)	47,176.34	31,066.34
PV(CFE at e)		31,066.34

The value of equity is the price that the owners would sell their participation in the firm and this is higher than the initial equity contribution of 24,000.

What happens when book values are used

In this section it is shown what conditions have to be met in order that calculations with book values coincide. This means that

$$PV(\text{FCF at } WACC_{\text{book values}}) = PV(\text{CFE at } e_{\text{book values}}) + PV(\text{CFD at } d)$$

The reason for this section is the widespread use of book value in the practice.

In Annex 3 and 4, it is shown that book values are valid for very restricted cases. In particular, in Annex 3, it is shown that book values are correct when the NPV of the firm is zero. When NPV is different to zero, then market value is mandatory.

Annex 4, shows the consequences and/or conditions resulting from the application of the valuation procedure with book values and constant WACC and with market values and constant WACC, with and without taxes. The standard procedure adopted by most practitioners is to calculate the WACC based on book values for year 0 or, in the best case, to calculate the WACC based on market values for year 0; the WACC is maintained constant through the time. Let us examine the situation of the firm value. First, book values are used to calculate WACC.

A summary of the different cases is presented in the next table

Table 11. Book value constant WACC

	Tax	No tax
$n > 1$		Holds for $D=0$ and/or $d=e=WACC$. Does not hold for $D > 0$ and/or e and d are not equal
$n=1$		Holds for $D=0$ and/or $d=e=WACC$
$n > 1$	$D=0$	
$n=1$	$D=0$ and/or $e=d(1-T)$	
Perpetuity		$D=0$ and/or $d=e=WACC$
Perpetuity	$D=0$ and/or $e=d(1-T)$	

Table 12. Market value, constant WACC

	Tax	No tax
$n > 1$		Holds for $D=0$ and/or $d=e=WACC$
$n=1$		Holds for $D=0$ and/or $d=e=WACC$
$n > 1$	$D=0$ Does not hold for $e=d(1-T)$ or $e=d$	
$N=1$	Holds for any value of D and any value of e, d and T	
Perpetuity		Holds for any value of D and any value of e, d and T
Perpetuity	Holds for any value of D and any value of e, d and T	

Tables 11 and 12 show that book value approach is valid for very restricted cases.

Conclusions

The misuse of WACC might be due to several reasons. Traditionally there have been no computing tools to solve the circularity problem in WACC calculations. Now it is possible and easy with the existence of spreadsheets. Not having these computing resources in the previous years it was

necessary to use simplifications such as calculating just one single discount rate or in the best of cases to use the book values in order to calculate the WACC.

Here a detailed (but known) methodology to calculate the WACC has been presented taken into account the market values in order to weigh the cost of debt and the cost of equity. By the same token a methodology based on the WACC before taxes ρ , constant (assuming stable macroeconomic variables, such as inflation) that does not depend on the capital structure of the firm has been presented, the tax savings and the capital structure of the firm. This methodology (TV WACC) takes into account the actual tax savings when they occur and the fact that usually losses are carried forward. The most difficult task is the estimation of ρ , or alternatively, the estimation of e . Here, a methodology to estimate those parameters is suggested.

An adjusted WACC (TV WACC) was presented. This formulation has the property to take into account the losses carried forward stating explicitly when tax savings are earned.

In summary, the different methodologies presented to calculate the total value of the firm are⁸:

Total Value for the firm $TV = PV(\text{FCF at Adjusted WACC})$

Total Value for the firm $TV = PV(\text{FCF at } \rho) + VP(\text{TS at } \rho)$

Total Value for the firm $TV = PV(\text{CCF at } \rho)$.

Market value of equity $E_{mv} = TV - D$

Market value of equity $E_{mv} = PV(\text{CFE at } e)$.

All these calculations coincide.

From tables 7 to 10 it can be seen how the different approaches to value calculation coincide, given that market values are used to calculate the appropriate discount rates for FCF and CFE and that the discount rate for TS is ρ .

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Annex 1

How to solve circularities in Excel

To solve circularities in Excel it is necessary to follow these instructions:

1. Select the option *Tools* in the textual menu in Excel.
2. Select *Options*
3. Select the tab *Calculate*.
4. In the dialog box select *Iteration* and click *Ok*.

This procedure can be done before starting the work in the spreadsheet or when Excel declares the presence of circularity.

It is expected that after this procedure the circularity is broken. However, it has been found that errors are displayed. In order to eliminate them, any number is written in the last WACC and that action is undone. The correct WACC will be displayed. This has to be repeated for every WACC until the solution is found.

The Practitioners' Point of View

Recently the authors were discussing about how people use and obtain the discount rate to appraise investment projects and to value firms. The authors were skeptical that practitioners would not take into account market values or any value at all. The authors decided to contact some of them. Some excerpts from the opinion of two financial officers are transcribed. One of them is from Latin America, working with a multi national firm, partner of a very important firm at the USA and they are developing an infrastructure project at Philippines. The other one is from a European firm with operations worldwide. In particular, the financial officer is posted at Japan.

The European executive said :

"1. Cost of equity (c) is fixed based on risk free interest (... Government bonds) + some adjustment to reflect risk (based on our impression of the risk).

2. Interest rate on debt (r) is the interest rate on debt we can obtain to a specific project in a specific country.

WACC is then calculated as:

$$r(1-\text{tax rate})D/E + (1-D/E)*c$$

where D/E is (book value)portion of financing done with debt and taxrate is the company tax rate in the country where the project is to take place. (I think at least this last point is rather questionable)."

The executive from Philippines said:

"With respect to WACC, I tell you what we do in practice. This obviously depend on the firm policies and on the person that makes the valuation., but in general, en 90% of the cases we do not pay too much attention to WACC.

The truth is that taking into account the number of variables that have to be considered in a valuation and that one has to project and that they could have a 100% variation, to try to be to fine on the

WACC is almost, I would say, a waste of time. For academic purposes it is great, but for practical purposes it is not. I know firms that take the debt cost and add a country risk if the investment is in a country different from the one where headquarters are located and THAT IS ALL, it is the cost of equity in order to discount the CFE. Simple, isn't it? And the firm I am mentioning is our partner in XYZ [*The name of the project at Philippines*] and they are the utility company with more investment overseas!!!!!! Billions and billions of dollars spent using this theory!!! For instance, they discounted the cash flows for our project at 12% (9% + 3% country risk), but the most beautiful of this case, is that they covered their dividends with an political risk insurance, and with this, I do not know what the 3% happens to be. Other firms, as we do when we are valuing a firm, try to put a little more brain to the issue but not too much to be "killed" by it, for the above mentioned reasons. I never have had a public [*traded at the Stock Exchange*] firm, and hence the information is not too good. For the cost of debt I take the cost those bond would cost if they traded them in the Exchange. It is possible that the firm has some bonds around, but they are not traded. For instance, ABCD [*the name of the company where he works*] has some bonds in USA, but they are not traded. If the firm does not have an outstanding debt, I simply would take the government bonds plus a spread that might be determined for the industry (there is always a similar firm that trades in the Exchange). For the equity, the ideal is to take what the industry asks (through beta) in countries with good data, for instance, the USA Exchange and make some adjustment to the country where you are making the valuation. But as I say, it is difficult to find people that go into too much detail in this. With respect to WACC (book value), if someone is working with WACC, the most probable situation is that she does not use book values, but use a proxy with information available although comparable and not with the firm information.

[...I

It is not easy to make generalizations of what is "in use in the market" when valuing a firm. What I do believe is that they don't pay too much attention to the WACC issue."

Annex 3

Let us demonstrate the problem in the single period case for two cases. It is assumed that the equality holds and then the WACC is derived from that assumption. Two cases will be shown. First the case without tax and second with tax.

The cash flows are:

Table A3 1. Cash flows

Year	0	1
Free cash flow	-K	$(R - K)(1 - T) + K$
Debt financing	D	$- D(1 + d)$
CFE	-E	$(R - K)(1 - T) + K - D(1 + d)$

K= Investment, R = Returns, K = Depreciation, T = Tax rate, D = Debt and d = Cost of debt before taxes.

Case 1:

Without tax, the traditional WACC formula is not correct with book values. The traditional formula will only be correct with market values.

Let R be the cash flow at the end of year 1 and K is the initial investment. Let ρ be the required return on unlevered equity. The depreciation L is equal to the initial investment K. The free cash flow is

$$FCF = (R - K) + K = R \tag{A3 1}$$

Now if it is accepted that MM holds, then in the case without tax, the return to unlevered equity ρ must be equal to the WACC.

PV of free cash flow

Then $PV(FCF) = \frac{R}{(1 + \tilde{r})}$ (A3 2a)

$$PV(FCF)(1 + \rho) = R \tag{A3 2b}$$

$$PV(FCF) + PV(FCF)\rho = R \quad (A3\ 2c)$$

$$PV(FCF) = R - PV(FCF)\rho \quad (A3\ 2d)$$

PV of debt

Let D be the amount of debt and E is the amount of equity, the initial book values. And the initial investment K is equal to the sum of the debt financing plus the equity contribution. That is, $K = D + E$.

Let the cost of debt be d. Then the payment for debt at the end of year 1 is $D(1 + d)$

$$PV(CFD) = \frac{D * (1+d)}{(1+d)} = D \quad (A3\ 3)$$

PV of CFE

$$\text{Cash Flow to Equity (CFE)} = (R - K)(1 - T) + K - D(1 + d) \quad (A3\ 4)$$

$$PV(CFE) = \frac{R - D(1+d)}{(1+e)} \quad (A3\ 5a)$$

$$PV(CFE)(1 + e) = R - D(1 + d) \quad (A3\ 5b)$$

$$PV(CFE) + PV(CFE)e = R - D(1 + d) \quad (A3\ 5c)$$

$$PV(CFE) = R - D(1 + d) - PV(CFE)e \quad (A3\ 5d)$$

Let us check if $PV(FCF) = PV(CFE) + PV(CFD)$. If each term of this equation is replaced by 12d, 15d and 13, then:

$$R - PV(FCF)\rho = R - D(1 + d) - PV(CFE)e + D \quad (A3\ 5e)$$

Simplifying, (subtracting R from both sides)

$$- PV(FCF)\rho = -Dd - PV(CFE)e \quad (A3\ 5f)$$

$$\tilde{n} = \frac{Dd + PV(CFE)e}{PV(FCF)} = dD\% + eE\% \quad (A3\ 5g)$$

As it was expected.

Now assume that

$$PV(FCF) = PV(CFE) + PV(CFD) \quad (A3\ 6a)$$

$$PV(FCF) = PV(CFE) + D \quad (A3\ 6b)$$

$$PV(FCF) - D = PV(CFE) \quad (A3\ 6c)$$

$$PV(FCF) - K + E = PV(CFE) \quad (A3\ 6d)$$

Let us calculate the NPV for the firm in this example. From A3 2d

$$PV(FCF) = \frac{R}{(1 + \ddot{o})} \quad (A3\ 2e)$$

$$PV(FCF)(1 + \text{Discount rate}) = R \quad (A3\ 2f)$$

$$PV(FCF) - PV(FCF)\phi = R \quad (A3\ 2g)$$

$$PV(FCF) = R - PV(FCF)(\phi) \quad (A3\ 2h)$$

$$PV(CFE) = R - D(1 + d) - PV(CFE)e \quad (A3\ 5d)$$

$$PV(FCF) - D = PV(CFE) \quad (A3\ 6c)$$

Combining line A3 5d and line A3 6c, then

$$PV(FCF) - D = R - D(1 + d) - PV(CFE)e \quad (A3\ 7a)$$

$$PV(FCF) = R - Dd - PV(CFE)e \quad (A3\ 7b)$$

Combining line A3 2h and line A3 7b, then

$$R - PV(FCF)\phi = R - Dd - PV(CFE)e \quad (A3\ 8a)$$

$$PV(FCF)\phi = Dd + PV(CFE)e \quad (A3\ 8b)$$

$$\ddot{o} = \frac{dD + PV(CFE)e}{PV(FCF)} = \tilde{n} \quad (A3\ 8c)$$

The NPV = PV(FCF) – K = PV(FCF) – D -E

When NPV = 0, then PV(FCF) = K = D + E. This means book value. Thus, in the zero NPV case, the traditional WACC formula holds. And the percentage of debt and the percentage of equity

are based on the book values and not on the market values. This should not be a surprise, because $NPV = 0$ implies the value of the firm is $K = D + E$, hence the market value is equal to the book value.

When NPV is not zero, the percentage of debt and equity are based on the market values, rather than the book values.

Case 2:

With taxes, the traditional WACC formula is not correct if it is assumed that the correct discount rate for the tax shield is d . Here the key idea is to determine the proper rate for the tax shield, and once this is done, then everything else falls into place.

Without tax, the traditional WACC formula is not correct with book values. The traditional formula will only be correct with market values.

Let R be the cash flow at the end of year 1 and K is the initial investment. Let ρ be the required return on unlevered equity. The depreciation L is equal to the initial investment K . Then the taxable income is $R - L = R - K$

The tax is $(R-K)*T$ and the net of tax, cash flow is

$$R - (R - K)*T = (R - K)*(1 - T) + K \tag{A3 9}$$

PV of free cash flow

Then
$$PV(FCF) = \frac{(R - K)(1 - T) + K - TdD}{(1 + \rho)} \tag{A3 10a}$$

$$PV(FCF)(1 + \rho) = (R - K)(1 - T) + K - TdD \tag{A3 11a}$$

$$PV(FCF) + PV(FCF)\rho = (R - K)(1 - T) + K - TdD \tag{A3 11b}$$

$$PV(FCF) = (R - K)(1 - T) + K - TdD - PV(FCF) \rho \tag{A3 11c}$$

PV of debt

Let D be the amount of debt and E is the amount of equity, the initial book value. And the initial investment K is equal to the sum of the debt financing plus the equity contribution. That is,

$$K = D + E \quad (A3\ 12)$$

Let the cost of debt be d . Then the payment for debt at the end of year 1 is $D(1 + d)$

$$PV(CFD) = \frac{D * (1 + d(1 - T))}{(1 + d(1 - T))} = D \quad (A3\ 13)$$

PV of CFE

$$\text{Cash Flow to Equity (CFE)} = (R - K)(1 - T) + K - D(1 + d) \quad (A3\ 14)$$

$$PV(CFE) = \frac{(R - K)(1 - T) + K - D(1 + d)}{(1 + e)} \quad (A3\ 15a)$$

$$PV(CFE)(1 + e) = (R - K)(1 - T) + K - D(1 + d) \quad (A3\ 15b)$$

$$PV(CFE) + PV(CFE)e = (R - K)(1 - T) + K - D(1 + d) \quad (A3\ 15c)$$

$$PV(CFE) = (R - K)(1 - T) + K - D(1 + d) - PV(CFE)e \quad (A3\ 15d)$$

Let us check if $PV(FCF) = PV(CFE) + PV(CFD)$. If each term of this equation is replaced by A3 11c, A3 15d and A3 3, then:

$$\begin{aligned} & (R - K)(1 - T) + K - TdD - PV(FCF) \phi \\ &= (R - K)(1 - T) + K - D(1 + d) - PV(CFE)e + D \end{aligned} \quad (A3\ 15e)$$

Simplifying, (subtracting $(R - K)(1 - T) + K$ from both sides)

$$- PV(FCF) \phi - TdD = -Dd - PV(CFE)e \quad (A3\ 15f)$$

$$\phi = \frac{Dd(1 - T) + PV(CFE)e}{PV(FCF)} = d(1 - T)D\% + eE\% = WACC \quad (A3\ 15g)$$

Now assume that

$$PV(FCF) = PV(CFE) + PV(CFD) \quad (A3\ 16a)$$

$$PV(FCF) = PV(CFE) + D \quad (A3\ 16b)$$

$$PV(FCF) - D = PV(CFE) \quad (A3\ 16c)$$

$$PV(FCF) - K + E = PV(CFE) \quad (A3\ 16d)$$

Let us calculate the NPV for the firm in this example. From above

$$PV(\text{FCF}) = (R - K)(1 - T) + K - TdD - PV(\text{FCF}) \phi \quad (\text{A3 11c})$$

$$PV(\text{CFE}) = (R - K)(1 - T) + K - TdD - (D(1 + d) - TdD) - PV(\text{CFE})e =$$

$$(R - K)(1 - T) + K - (D(1 + d) - PV(\text{CFE})e) \quad (\text{A3 15d})$$

$$PV(\text{FCF}) - D = PV(\text{CFE}) \quad (\text{A3 16c})$$

Combining line A3 15d and line A3 16c, then

$$PV(\text{FCF}) - D = (R - K)(1 - T) + K - D(1 + d) - PV(\text{CFE})e \quad (\text{A3 17a})$$

$$PV(\text{FCF}) = (R - K)(1 - T) + K - Dd - PV(\text{CFE})e \quad (\text{A3 17b})$$

Combining line A3 11c and line A3 17b, then

$$(R - K)(1 - T) + K - TdD - PV(\text{FCF}) \phi =$$

$$(R - K)(1 - T) + K - Dd - PV(\text{CFE})e \quad (\text{A3 18a})$$

$$PV(\text{FCF})(\text{Discount rate}) = Dd - TdD + PV(\text{CFE})e \quad (\text{A3 18b})$$

$$\phi = \frac{dD(1 - T) + PV(\text{CFE})e}{PV(\text{FCF})} = \text{WACC} \quad (\text{A3 18c})$$

The NPV = PV(FCF) – K = PV(FCF) – D - E

When NPV = 0, then PV(FCF) = K = D + E. This means book value. Thus, in the zero NPV case, the traditional WACC formula holds. And the percentage of debt and the percentage of equity are based on the book values and not on the market values. This should not be a surprise, because NPV = 0 implies the value of the firm is K = D + E, hence the market value is equal to the book value.

When NPV is not zero, the percentage of debt and equity are based on the market values, rather than the book values.

Annex 4

In this section it is shown what conditions have to be met in order that calculations with book values coincide. This means that

$$PV(\text{FCF at } WACC_{\text{book values}}) = PV(\text{CFE at } e_{\text{book values}}) + PV(\text{CFD at } d)$$

The reason for this annex is the widespread use of book value in the practice.

Cases

In these cases the consequences and/or conditions resulting from the application of the valuation procedure with book values and constant WACC and with market values and constant WACC, with and without taxes are shown. The standard procedure adopted by most practitioners is to calculate the WACC based on book values for year 0 or, in the best case, to calculate the WACC based on market values for year 0; the WACC is maintained constant through the time. Let us examine the situation of the firm value. First, book values are used to calculate WACC.

A summary of the different cases is presented in the next table

Table 13. Book value constant WACC

	Tax	No tax
n>1		Holds for D=0 and/or d=e=WACC. Does not hold for D>0 and/or e and d are not equal
n=1		Holds for D=0 and/or d=e=WACC
n>1	D=0	
n=1	D=0 and/or e=d(1-T)	
Perpetuity		D=0 and/or d=e=WACC
Perpetuity	D=0 and/or e=d(1-T)	

Table A4 1. Market value, constant WACC

	Tax	No tax
n>1		Holds for D=0 and/or d=e=WACC
n=1		Holds for D=0 and/or d=e=WACC
n>1	D=0 Does not hold for e=d(1-T) or e=d	
N=1	Holds for any value of D and any value of e, d and T	
Perpetuity		Holds for any value of D and any value of e, d and T
Perpetuity	Holds for any value of D and any value of e, d and T	

Case 1A Book values. No tax. Cash flows $F_i = 0$ for $i = 1, 2, \dots, n-1$ and $F_n = F$

Assume that debt is D and equity is $(1-D)$. Assume that cost of debt is d and cost of equity is e .

Cash flow to debt CFD and cash flow to equity CFE at period n are as follows:

Table A4 2. N period example with book values

Period	FCF	CFD	CFE
n	F	$D(1+d)^n$	$F-D(1+d)^n$

Weighted Average Cost of Capital WACC is $\alpha d + (1-\alpha)e$. Present values for each cash flow are:

$$PV(FCF) = \frac{F}{(1 + Dd + (1 - D)e)^n} \quad (A4\ 1)$$

$$PV(CFD) = \frac{D(1+d)^n}{(1+d)^n} = D \quad (A4\ 2)$$

$$PV(CFE) = \frac{F - D(1+d)^n}{(1+e)^n} \quad (A4\ 3)$$

The total value of the firm calculated using FCF is (sequential valuation procedure):

$$PV(FCF) = \frac{F}{(1 + Dd + (1 - D)e)^n} \quad (\text{A4 4})$$

The total value of the firm calculated using CFE is

$$PV(CFE) + D = \frac{F - D(1 + d)^n}{(1 + e)^n} + D \quad (\text{A4 5})$$

For $PV(CFE) + D = PV(FCF)$ this relation should hold:

$$\frac{F}{(1 + Dd + (1 - D)e)^n} = \frac{F - D(1 + d)^n + D(1 + e)^n}{(1 + e)^n} \quad (\text{A4 6})$$

Rearranging terms,

$$\frac{F}{(1 + D(d - e) + e)^n} = \frac{F + D((1 + e)^n - d(1 + d)^n)}{(1 + e)^n} \quad (\text{A4 7a})$$

(A4 7a) holds when $D = 0$ or when $e = d$ or both. The equation (A4 7a) is transformed to

$$\frac{F}{(1 + e)^n} = \frac{F}{(1 + e)^n} \quad (\text{A4 7b})$$

When $n = 1$

$$\frac{F}{((1 + e) - D(e - d))} = \frac{F + D(e - d)}{(1 + e)} \quad (\text{A4 8})$$

The only case when this equality holds is when $\alpha(e - d) = 0$ and this only happens when $d = e =$
WACC or $\alpha = 0$.

In general eq. (A4 8) does not hold for $n > 1$.

Case 1B Book value. Tax = T Book values Total investment 1 debt = D, E = 1-D

$$FCF = F; \quad CFD = D(1 + d)^n; \quad CFE = F - D(1 + d)^n + (D(1 + d)^n - D)T \quad (\text{A4 9})$$

$$WACC = Dd(1 - T) + (1 - D)e \quad (\text{A4 10})$$

$$PV(FCF) = \frac{F}{(1 + Dd(1 - T) + (1 - D)e)^n} \quad (\text{A4 11})$$

$$PV(CFE) + D = \frac{F - D(1+d)^n + (D(1+d)^n - D)T}{(1+e)^n} + D \quad (A4\ 12)$$

$$PV(CFE) + D = \frac{F - D(1+d)^n + (D(1+d)^n - D)T + D(1+e)^n}{(1+e)^n} \quad (52a)$$

$$PV(CFE) + D = \frac{F + D(- (1+d)^n + (1+d)^n T - T + (1+e)^n)}{(1+e)^n} \quad (A4\ 12b)$$

$$PV(CFE) + D = \frac{F + D((1+e)^n - (1+d)^n (1-T) - T)}{(1+e)^n} \quad (A4\ 12c)$$

$$PV(CFE) + D = PV(FCF) = \frac{F}{(1+Dd(1-T) + (1-D)e)^n} = \frac{F + D((1+e)^n - (1+d)^n (1-T) - T)}{(1+e)^n} \quad (A4\ 12d)$$

This last equation holds when $D=0$ and its value is $\frac{F}{(1+e)^n}$

When $n=1$,

$$PV(CFE) + D = PV(FCF) = \frac{F}{(1+Dd(1-T) + (1-D)e)} = \frac{F + D((1+e) - (1+d)(1-T) - T)}{(1+e)}$$

(A4 12e)

This equation holds when $D=0$ or when $d(1-T) = e = WACC$ or both and its value is $\frac{F}{(1+e)}$

Case 2 Book values. Assume that debt is D and equity is $(1-D)$. Assume that cost of debt is d and cost of equity is e . Cash flows are a perpetuity equal to F and the loan is a perpetuity equal to D . Cash flow to debt CFD and cash flow to equity CFE at period n are as follows:

Table A4 3. Basic assumptions for perpetuity

Period	FCF	CFD	CFE
1 to ∞	F	Dd	$F-Dd$

Weighted Average Cost of Capital WACC is $D\%d + (1-D)\%e$. Present values for each cash flow are:

$$PV(FCF) = \frac{F}{Dd + (1-D)e} \quad (A4\ 13)$$

$$PV(CFE) + D = \frac{F - d}{e} + D \quad (A4\ 14)$$

$$\frac{F}{e - D(e - d)} = \frac{F + D(e - d)}{e} \quad (A4\ 15)$$

The only case when this equality holds is when $\alpha(e-d) = 0$ and this only happens when $d = e =$ WACC or $D = 0$.

Case 2A Tax = T Book values Perpetuity

$$FCF = F; \text{CFD} = Dd \text{ and } CFE = F - Dd(1-T) \quad (A4\ 16)$$

$$WACC = Dd(1-T) + (1-D)e \quad (A4\ 17)$$

$$PV(FCF) = \frac{F}{Dd(1-T) + (1-D)e} = \frac{F}{e + D(d(1-T) - e)} \quad (A4\ 18a)$$

$$PV(CFE) + D = \frac{F - Dd(1-T)}{e} + D = \frac{F - Dd(1-T) + De}{e} = \frac{F - D(d(1-T) - e)}{e} \quad (A4\ 18b)$$

$PV(FCF) = PV(CFE) + D$ holds when $D(d(1-T) - e) = 0$ and this happens when $D = 0$ or $d(1-T) = e$ or both.

Example 1

Let us try with some figures. For $D = 0.70$, after tax debt rate, $d = 25\%$, cost of equity $e = 25\%$, WACC = 25%, $n = 1$ and $FCF = 1.3$ then

$$PV(FCF) = \frac{1.3}{(1 + Dd + (1-D)e)} - D = 1.04$$

$$PV(CFE) = \frac{1.3 - D(1+d)}{(1+e)} = 1.04$$

The relationship between the two values should be 1.

Using *Table* from Excel, it is possible to examine the behavior of the relationship between the FCF-calculated value and the CFE-calculated value. Assuming constant WACC (a MM world), then

Table A4 4 Value FCF/Value CFE

	d						
	0%	2.50%	5%	10.00%	15%	25%	
Leverage	0%	1.000	1.000	1.000	1.000	1.000	1.000
5%	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10%	1.001	1.001	1.001	1.001	1.001	1.000	1.000
15%	1.001	1.001	1.001	1.001	1.001	1.001	1.000
20%	1.002	1.002	1.001	1.001	1.001	1.001	1.000
25%	1.002	1.002	1.002	1.001	1.001	1.001	1.000
30%	1.003	1.003	1.002	1.002	1.001	1.001	1.000
35%	1.004	1.003	1.003	1.002	1.002	1.000	1.000
40%	1.005	1.004	1.004	1.003	1.002	1.000	1.000
45%	1.005	1.005	1.004	1.003	1.002	1.000	1.000
50%	1.006	1.006	1.005	1.004	1.003	1.000	1.000
55%	1.008	1.007	1.006	1.005	1.003	1.000	1.000
60%	1.009	1.008	1.008	1.006	1.004	1.000	1.000
65%	1.011	1.010	1.009	1.007	1.005	1.000	1.000
70%	1.012	1.012	1.011	1.008	1.006	1.000	1.000
75%	1.015	1.014	1.013	1.010	1.008	1.000	1.000
80%	1.017	1.016	1.015	1.013	1.009	1.000	1.000
85%	1.021	1.020	1.019	1.016	1.012	1.000	1.000
90%	1.025	1.024	1.023	1.020	1.016	1.000	1.000
95%	1.031	1.031	1.030	1.027	1.024	1.000	1.000

Given that cash flows might be infinite, it is not easy to make an analytical proof for period $n > 1$. However, one might approach this problem from a practical point of view. Using Goal Seek from Excel, one can find when a given condition is met.

Example 2¹⁰

If a cash flow is set as follows and $D=70\%$, after taxes debt rate $d = 20\%$, cost of equity $e = 20\%$, and *constant* WACC = 20%, then

Table A4 5. Cash flows for example

	FCF	CFD	CFE
0	-1,000	-700	-300
1	450	140	310
2	500	140	360
3	350	140	210
4	400	140	260
5	1,450	840	610
NPV	\$700.39	\$0.00	\$700.39

Using *Table* from Excel, it is possible to examine the behavior of the relationship between the firm value calculated as $PV(\text{FCF}, \text{WACC})$ and the firm value calculated as $PV(\text{CFE}, e)$. Assuming a *variable* WACC, then

Table A4 6 Value FCF/Value CFE variable WACC

	d									
	1.000	0.0%	2.5%	5.0%	7.5%	10.0%	12.00%	15%	17.50%	20%
Leverage	0%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5%	1.022	1.019	1.016	1.013	1.011	1.008	1.005	1.003	1.000
	10%	1.044	1.038	1.033	1.027	1.022	1.016	1.011	1.005	1.000
	15%	1.067	1.058	1.050	1.041	1.033	1.024	1.016	1.008	1.000
	20%	1.091	1.079	1.067	1.055	1.044	1.033	1.022	1.011	1.000
	25%	1.117	1.101	1.085	1.070	1.055	1.041	1.027	1.013	1.000
	30%	1.143	1.123	1.104	1.085	1.067	1.050	1.033	1.016	1.000
	35%	1.170	1.146	1.123	1.101	1.079	1.058	1.038	1.019	1.000
	40%	1.199	1.170	1.143	1.117	1.091	1.067	1.044	1.022	1.000
	45%	1.229	1.195	1.163	1.133	1.104	1.076	1.050	1.024	1.000
	50%	1.260	1.221	1.184	1.150	1.117	1.085	1.055	1.027	1.000
	55%	1.292	1.248	1.206	1.167	1.130	1.095	1.061	1.030	1.000
	60%	1.325	1.275	1.229	1.184	1.143	1.104	1.067	1.033	1.000
	65%	1.361	1.304	1.252	1.202	1.157	1.113	1.073	1.035	1.000
	70%	1.397	1.334	1.275	1.221	1.170	1.123	1.079	1.038	1.000
	75%	1.436	1.365	1.300	1.240	1.184	1.133	1.085	1.041	1.000
	80%	1.476	1.397	1.325	1.260	1.199	1.143	1.091	1.044	1.000
	85%	1.518	1.431	1.352	1.280	1.214	1.153	1.098	1.047	1.000
90%	1.562	1.466	1.379	1.300	1.229	1.163	1.104	1.050	1.000	
95%	1.608	1.502	1.407	1.321	1.244	1.174	1.110	1.053	1.000	

Example 3¹¹

If a cash flow is set as follows and $\alpha=70\%$, debt rate, after taxes debt rate $d = 20\%$, cost of equity $e = 20\%$, and constant WACC = 20%, then

Table A4 7. Cash flows for the example.

	FCF	CFD	CFE
0	-1,000	-700	-300
1	450	140	310
2	500	140	360
3	350	140	210
4	400	140	260
5	1,450	840	610
NPV	\$700.39	\$0,00	\$700,39

Using *Table* from Excel, it is possible to examine the behavior of the relationship between the firm value calculated as PV(FCF, WACC) and the firm value calculated as PV(CFE, e). Assuming a *constant* WACC, then

Table A4 8 Value FCF/Value CFE constant WACC

	d									
	1.000	0.0%	2.5%	5.0%	7.5%	10.0%	12.5%	15.0%	17.5%	20.0%
Leverage	0%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5%	1.022	1.019	1.017	1.014	1.011	1.008	1.006	1.003	1.000
	10%	1.047	1.041	1.035	1.029	1.024	1.018	1.012	1.006	1.000
	15%	1.075	1.066	1.056	1.047	1.037	1.028	1.019	1.009	1.000
	20%	1.107	1.094	1.080	1.067	1.053	1.040	1.026	1.013	1.000
	25%	1.143	1.125	1.107	1.089	1.071	1.053	1.035	1.018	1.000
	30%	1.185	1.162	1.138	1.115	1.092	1.068	1.046	1.023	1.000
	35%	1.234	1.204	1.174	1.145	1.115	1.086	1.057	1.029	1.000
	40%	1.292	1.254	1.217	1.180	1.143	1.107	1.071	1.035	1.000
	45%	1.361	1.314	1.268	1.222	1.177	1.132	1.087	1.043	1.000
	50%	1.445	1.387	1.330	1.273	1.217	1.162	1.107	1.053	1.000
	55%	1.548	1.477	1.406	1.336	1.267	1.199	1.131	1.065	1.000
	60%	1.680	1.591	1.503	1.416	1.330	1.245	1.162	1.080	1.000
	65%	1.852	1.740	1.629	1.519	1.412	1.305	1.201	1.099	1.000
	70%	2.086	1.942	1.800	1.660	1.522	1.387	1.254	1.125	1.000
	75%	2.419	2.231	2.045	1.861	1.680	1.503	1.330	1.162	1.000
	80%	2.930	2.673	2.419	2.169	1.922	1.680	1.445	1.217	1.000
	85%	3.798	3.427	3.059	2.695	2.335	1.983	1.640	1.311	1.000
90%	5.571	4.969	4.370	3.776	3.188	2.610	2.045	1.503	1.000	
95%	11.012	9.710	8.413	7.122	5.840	4.569	3.318	2.107	1.000	

¹ Part of this paper was developed while Mr. Velez-Pareja was teaching at Universidad Javeriana, Bogotá, Colombia.

² From now on the value of debt and the value of equity will be referred to as D and E, respectively. The debt and equity as a percent of total levered value will be denoted by D% and E%.

³ They assume that except number 1 period the proportion of debt is constant. In other words, they assume that for period 1 the correct discount rate for the TS is d and for the rest of periods is ρ .

⁴ This information is based on real data for real nominal risk free rates in the Colombian bond market.

⁵ If ρ is used directly to calculate the total value of the firm, circularity will not be found. However, when the comparisons between the different approaches are made, when traditional WACC is used it is unavoidable to find circularity, because the procedure implies to use WACC for calculating total value and not ρ .

⁶ In fact, in the article the authors say that the methodology is to calculate the risk of the cost of capital, although at the end they say it is to define the risk for the equity cost. The way the methodology is presented allows thinking that it is the firm risk that is dealt with and this risk is added to the risk free rate. With this the cost of capital before taxes for the firm is found. This would be ρ

⁷ As MM say that ρ is constant and independent from the capital structure, it will be equal to ρ when debt is zero. This ρ is WACC before taxes. And this is the condition for the validity of the first proposition of MM.

⁸ There exist others methodologies, but they do not coincide among them. See Taggart, 1991

⁹ The debt valuation is done either by the contractual conditions or the market value. In this note the contractual value is the value obtained by discounting the future cash flows of debt

at the cost of debt after taxes. This method gives the actual value of debt (the actual balance).

¹⁰ This is an illustration for the previous findings. The cost of equity e must be higher than d . The values used here are to illustrate the case when consistency is found using book values. It is not possible in a MM world.

¹¹ This is an illustration for the previous findings. The cost of equity e must be higher than d . The values used here are to illustrate the case when consistency is found using book values. It is not possible in a MM world.