MATHEMATICS AS A TOOL OF ACCOUNTING INSTRUCTION AND RESEARCH

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It is very difficult as well as unsatisfying to speak on the uses of mathematics in accounting for two main reasons. Firstly, because discussions of this nature convey the somewhat false impression that the only impetus toward changes—if any progress is evident in the use of mathematics in accounting instruction and research—has originated from without rather than from within the accounting profession, and that progress has been forced upon the accounting discipline by outsiders. Secondly, because the potential uses of mathematics in accounting are so many, within the time limitations of a meeting one can at best only survey the area, in other words sketch a superficial picture with no depth or detail. This feeling of uneasiness and dissatisfaction, however, does not reflect on the usefulness of discussions such as this, but rather reveals some type of helplessness and inferiority complex if you please. Here we have mathematical theory, tools, and techniques some of which have been developed centuries ago, and not only did we just recently come to realize their relevance to accounting, but furthermore our involvement is still for the most part in the realm of aesthetic appreciation.

I shall not attempt to explore fully the reasons we have not taken advantage of the existing body of mathematical knowledge earlier, but I cannot help speculating briefly on this issue. It appears to me that the demands of management for new quantitative criteria of efficiency of operations and decisions both aggregative and partial, are presenting opportunities and pressures that accounting cannot ignore. While in the past the accounting profession has always made valiant efforts to meet the challenges thrusted upon it, yet the truth is that it has been reluctantly following and not leading. Unless we exhibit more insight and vigor in developing and applying new methods that will help us understand and instruct others on the dynamics of the incessant process of resource allocation, we will find ourselves preaching and professing things that are no longer vital for managerial decision making. The signs are here, and if we do not take more initiative, the rapidly multiplying teams of the so-called "operations analysts" will subordinate the accountant to a custodian of dead files. In a more hopeful vein, irrespective of whether this will be the result of the work of accountants, operations researchers, economists, systems analysts, mathematicians, or all combined, a breakthrough in the methods of measurement cannot help but come in the late sixties or early seventies. The accountant because of his present functions is in a key position to explore as well as exploit the opportunities afforded by such a technological breakthrough, and become the head of a vast communications network that will provide management with the tools and information that are necessary for internal control, planning and decision making.

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Frankly, I am not attempting to advance or perpetuate the hegemony of accountants, because I am not so much concerned about who will be the custodian of such an integrated information network. My only concern is that this be done most efficiently and expeditiously. Of all corporate officials, however, the chief accounting officer, by the nature of his present duties, supervises communications over the greatest part of the area that is of managerial concern. He is able to observe the interactions of the various operational subunits of the firm and influence the firm's organizational structure. Through the years of practice, he has accumulated knowledge that can be of immense value to anyone who is interested in designing a comprehensive information system for managerial control and decision making and in general motivating efficiency for the accomplishment of the overall objectives of the firm. The only drawback is that the many years of practice that helped the accountants develop and solidify professional knowledge, have also generated some inertia toward radical changes and given reasons to some people to question the nature of the accountants' mentality. Appreciating the advantages that the accountant has in this respect over anyone else, because of his position and accumulated knowledge, and furthermore being an optimist I believe that the greatest pay-off for the firm lies in the education of the accountant to master the intricacies of his future duties rather than in his substitution. But it is here where the challenge comes. No one will serve knowledge to the accountant on a golden platter. He must attempt to acquire it by himself. The changes must come from within before it is too late for the accountant to benefit either himself or his firm.

Mathematics can be used in two major ways in accounting teaching and research. It can be used either as an *operational or optimizing tool* or as a *tool for building models* that purport to describe operations. While it is true that no such clear-cut classification can be made in many cases, yet because of the greater role that behavioral constraints assume in model building, it is convenient for expositional purposes to distinguish between these two rather artificial classifications.

Use of mathematics as an operational tool. The greatest effort as well as measurable success has so far come from areas where mathematics has been used as an operational tool. Some applications such as for inventory control, equipment replacement formulations or the derivation of flexible budgets and linear regression forecasts have been in use, admittedly often crudely but nonetheless used, for so long and are so well known (even though not universally) that I shall not burden you with any elaboration. More recently, as can be attested by a glance at the professional literature,[1] a lot of efforts have been devoted to the use of statistical sampling

[1] Cyert, R. M., G. M. Hinkley, and R. J. Monteverde, "Statistical Sampling in the Audit of the Air Force Motor Vehicle Inventory," The Accounting Review, Vol. XXXV, No. 4, October 1960, pp. 667–73.

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Neal, Dewey W., "Cost Control Charts—An Application of Statistical Techniques," *NAA Bulletin*, Vol. XLII, No. 9, May 1961, pp. 73–78.

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Vance, Lawrence L., "A Review of Developments in Statistical Sampling for Accountants," The Accounting Review, Vol. XXXV, No. 1, January 1960, pp. 19.

Vance, L. L. and Neter, John, *Statistical Sampling for Auditors and Accountants*, John Wiley & Sons, Inc., New York, 1956.

Trueblood, Robert M., and Cyert, Richard M., *Sampling Techniques in Accounting*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1957.

Cyert, R. M., and Davidson, H. Justin, *Statistical Sampling for Accounting Information*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962.

Hill, Henry P., Roth, Joseph L., and Arkin, Herbert, *Sampling in Auditing*, The Ronald Press Company, New York, 1962.

Arkin, Herbert, *Handbook of Sampling for Auditing and Accounting*, McGraw-Hill Book Company, Inc., New York, 1963.

techniques for cost control, inventory valuation, and account verification, thus pointing out that the accountant has finally abandoned or at least has come to question the false security of elusive deterministic solutions. In my opinion this is a good trend which will hopefully lead to the elimination of a lot of repetitive bookkeeping records, especially in the area of tracing the costs of the manufacturing product. There seems to be little justification in the present practice of tracing the standard cost of production into subsidiary manufacturing accounts in cases where the operations are repetitive. A lot of bookkeeping costs can be saved with no loss in control if random sampling techniques are applied for variance analysis. As it was pointed out on another occasion,[2] we must not forget that normalization of costs and prediction imply stability in expected values or averages but not in terms of particular observations. Consequently, efforts to ascribe significance to each and every particular deviation from normal are self-defeating, and emanate from the misguided belief that we live in a world of certainty where there can be no deviations attributable to purely stochastic elements.

As everyone knows, a "standard" is an average, or expected value. In the process of developing standards a lot of assumptions are made concerning "normal technology," "normal efficiency of inputs," "normal quality," "normal value of activity," "normal prices," for "the normalizing accounting period." Predictive standards, of course, can be either good, in the sense that they are representative of average "actual" costs, or bad. In either case, however, I hope that I can present convincing arguments against the practice of tracing the manufacturing costs into the various "work-in-process" accounts. Putting my contention in more emphatic terms, I believe that most if not all subsidiary accounts between the "raw materials" and the "finished goods" stage as well as the ancillary records that back them up can be abolished without any loss of control, if standard costs can be developed for the finished product.[3] In terms of cost, this implies a lot of bookkeeping savings but more important it means that management will no longer have to plough through an enormous amount of meaningless accounting variances, wasting precious time trying first to understand what these represent and then ascribing to them distorted import where in most cases there is none. More often than not the recipients of these accounting documents not knowing what is the nature of the information conveyed, do not even bother to look at the reports after their first exposure to them. Occurrences of this sort should not make the accountant glow with satisfaction, because the existence of confusion is not an indication of the importance and indispensability of the accounting function. On the contrary if the records can help no one in his efforts to perform his duties better, either because the reports are too complicated or meaningless, then such records should be abolished.

Turning to the reasons behind the contention that I have just presented, let us first take the case where the standards can be trusted as representing "actual" operations, given, of course, the particular company policies concerning cost allocations. Whenever we say that standards are valid, we mean that *over the normalizing period* average "actual" costs should approximate the predetermined standard costs. Consequently the predictability of standard costs cannot be tested over the shortrun but only over the normalizing period. What use then can we make of

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[2] See Zannetos, Zenon S., "Statistical Attributes of Group Depreciation" THE ACCOUNTING REVIEW, Vol. XXXVII, No. 4, October 1962, pp. 713–720.

[3] I am not speaking here about systems used for developing piece incentive payrolls.

these standards for control purposes? Since we have assumed in this case that the standards are valid, we can use them for testing probabilistically whether our process is out of control. But for the latter test we do not need to question each and every observation.[4] We can sample test, thus achieving substantial bookkeeping cost savings and still be as well off, if not better, from the control point of view as before.[5] Under a sample testing system the processing of costs can be much faster because it will be selective, thus providing management with more timely information. Furthermore, the practices that are suggested here will emphasize probabilistic *analysis* rather than deterministic listing of figures with consequent better information for decision making. If now the standards are not representative of "normal" operations and therefore cannot be trusted, then how can these be used as a guidepost for evaluating performance?

In summary then, we can say that whether the standards are valid or not the answer to questions concerning the tracing of costs into manufacturing accounts is the same. Whenever operations are repetitive and standardizable the value of continuous perfect information is very little, assuming that perfect information can be obtained. Control can be effectively exercised by means of sample testing thus resulting in faster feed-back as well as substantial bookkeeping cost savings. For inventory valuation and income determination it is sufficient to collect aggregate figures[6] and resort to the well-established practice of "standard equivalent" for obtaining the closing work-in-process figure. Consequently those of us who are in the academic field teaching accounting should start contemplating changes in our curricula to incorporate probabilistic notions and initiate research projects in order to develop the particular techniques for their practical implementation.

Leaving the area of sampling, we find that the theory developed in conjunction with Markov chains has exciting potential uses in the area of accounting both on the operational as well as conceptual levels. Cyert, Davidson and Thompson of the Carnegie Institute of Technology have been conducting research in the uses of Markov chains for estimating the allowance for doubtful accounts in retail establishments with very encouraging results.[7] On the same lines but on a much less ambitious scale, I have been studying independently the feasibility of using Markov chains in conjunction with group depreciation, and have reached the conclusion that the use of Markov chains will be very advantageous in providing data on the economic life of assets and the economic processes of obsolescence.[8] An attempt to use compound chains for explaining the liquidity preference of firms has not been very successful from the operational point of view, but I am not yet convinced that it cannot be done.

It appears that the theory connected with Markov processes may one day help us in determining the "optimal" size of the various accounts that appear on corporate Balance Sheets, but a lot of research will be necessitated between now and that happy day. The notion of optimality as used here should not be taken to imply the existence of an "optimum optimorum," because I do not believe that the latter is an attainable goal. What we will achieve, I hope, is an understanding of the inter-

[4] This has been recognized a long time ago by people involved in quality control.

[5] Under process costing the problem of data collection is not as extensive as it is in the case of job-order costing. For control purposes, however, even in the case of process costing we need to collect appropriate data "by product by process," unless the firm manufactures one homogeneous product.

[6] The degree of aggregation will be a function of the control aspects of the organizational structure of the firm.

[7] O.N.R. Research Memorandum No. 81.

[8] See "Statistical Attributes of Group Depreciation," op. cit., fn. 2.

relationships that exist between the various "productive," "enabling," "liability" and "owner's equity" accounts and, by using such an understanding, determine probabilistically the impact of different types of policies on the objectives of the firm. Given that we will be dealing with multi-dimensional vector spaces, we cannot hope to consider or exhaust all of the possible (multifold infinity of) alternatives, consequently we will always be striving for better and better feasible solutions, and hopefully always making progress.

The potential uses of statistics are so great, that one can devote many sessions such as this elaborating on this topic. Consequently within the time allotted for this meeting, I will limit myself to only mentioning the uses of Bayesian Statistics in decision theory and testing hypotheses, hoping that such a fleeting and superficial discussion will still suffice in giving you an appreciation of the vast horizons that modern statistical decision theory has now opened for us.

When we were discussing the use of sampling techniques no mention was made of the size of the sample or of the value of the information obtained. It is obvious that the segregation of deviations is not the end result of the accounting processes. And even if we were to agree that the accountant should not interpret but only report the results of operations, still the question of what is worth reporting and in what degree of refinement remains unresolved. Unless of course we wish to eliminate embarrassing questions of this nature by subordinating the accountant to the position of a mere bookkeeper who is told what to do and knows how to do it but knows none of the reasons justifying what he is doing.

Luckily most, if not all, of use here have a more ambitious notion of the functions of the accountant. Although we may disagree on what is encompassed by the principle of "objectivity" and what consequences emanate therefrom, we would still include, I trust, among the functions of the accountant the design of the accounting system. Consequently, as accountants we cannot but get involved in issues concerning the value of the information obtained, especially since information is neither readily available nor free.[9] Furthermore, since we live in a world of uncertainty, the result of an experiment (an observation) will not give us with certainty the true "state of nature" even if the latter is stable.

At present most of the decisions on issues of the aforementioned nature are either made by default or by the use of intuition. Of these two methods the only one of course that has any claim to validity is the one based on intuition. The latter, however, is often irrational, faulty and inconsistent, and since it is used only implicitly it provides no opportunity for learning. It is here that modern statistical decision theory can aid in combining optimizing tools and logic with intuition. Under statistical decision rules one is first of all forced to enumerate all the possible outcomes of his experiment S_i and specify all the alternative decisions $d(S_i)$ that he can make, given the various outcomes. Then he proceeds to assign a numerical loss[10] (loss function) $L(\theta, \alpha_i)$ that will be incurred if the true parameter (characteristic) of the population with which he is dealing is θ and the action taken is α_i. The latter action is a function of the various decisions d_K given S_i, or in notation $\alpha_i = \alpha_i(d_K | S_i)$. Finally the expected loss or risk that one undertakes by implementing decision d is given by:

$$\rho(\theta, d) = \sum_i L(\theta, d(S_i)) P_\theta(S_i)$$

[9] Actually the implication here is that the accountant should be also concerned with the cost of reporting and the consequent cost of reviewing the reported data by the recipients of the accounting reports.

[10] Of course a negative loss will imply a gain.

where $P_\theta(S_i)$ stands for the probability of the outcome S_i assuming that the state of nature is θ. What the above equation really gives us is the expected loss that results from a policy decision d which dictates the specific actions α_i that we should take as soon as we observe S_i. Of course all this assuming that θ is the true state of nature.

The determination of the loss function that we have just described is very important because it can include many subjective items (weights), that cannot be incorporated in profits and losses as normally defined. In addition if there exists asymmetry in the importance of equivalent deviations from given objectives, depending on their position in the range spectrum, the loss function is indispensable.

So far we have not actually forced the account to penetrate too much into the realm of normative judgment, because he often performs in a very simplified form this exercise implicitly while using his intuition. There is evidence, however, that in his quest for exact or actual cost as well as in his use of standards he selects as certain one state of nature while ignoring others. Furthermore, certain aspects of the principle of objectivity that were mainly intended for financial reporting purposes, and not for system design or decision making, have permeated all other aspects of the accountant's activities and resulted in deterministic regimentation.

We can now easily extend the expected loss function to cover cases where the states of nature θ_j are unknown. By introducing a subjective probabilistic (discrete) distribution for the various alternative states of nature $P(\theta=\theta_j)$ in the above formulation, we can get the expected risk, which is:

$$E(\rho(\theta_j, d)) = \sum_j \sum_i L(\theta_j, d(S_i)) P_{\theta_j}(S_i) P(\theta = \theta_j)$$

The famous Bayesian decision principle dictates a policy decision d_K that minimizes the above function, if it represents cost, and maximizes if it represents value. By testing therefore all the alternative decisions we can find the one which minimizes the expected risk and thus arrive at the optimal decision according to the Bayesian principle.[11]

There is another result of Bayes' work that we can use very effectively. As we proceed with our day to day operations and acquire new knowledge (obtain new observations) we can use the new empirical information in order to revise our subjective probabilities. This is in effect the essence of learning and can be continued *ad infinitum.* If for example we have observed S' and wish to test if θ is true given S', we can use Bayes' rule to find the conditional probability $P(\theta|S')$. Then suppose that we have obtained S'' and wish to use the information for another revision, we can use the probabilities sequentially, namely find the probability $P(\theta|S', S'')$ that θ is true given S' and then S''.[12] It is probably by now obvious that Bayes' rule can be used for testing hypotheses, and checking on the validity of established standards. For account-control and decision making, therefore, the value of Bayesian statistics cannot be ignored.

I do not know whether the above arguments have caused in some people a certain degree of apprehension due to the introduction of subjective value judgments. While such a concern may not be unnatural yet it is not completely justified. For the lot of the modern accountant, whether

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[11] This is not the only principle that one can apply, however. Among others we have the minimax, modified minimax and minimax regret principles. For a brief but sufficient description of these principles see: B. W. Lindgren and G. W. McElrath, *Introduction to Probability and Statistics*, The Macmillan Company, New York, 1959, pp. 243–246.

[12] For an illustrative example see: Alan S. Manne, *Economic Analysis for Business Decisions*, McGraw-Hill Book Company, Inc., New York 1961, pp. 155–167.

we like it or not—and we better like it if we know what is good for us—is not devoid of value judgments, especially in the design of the accounting system itself. While it may be true that many parts of present accounting systems are perpetuated by default, it is time for us to stop and question our own internal efficiency. If we do that we will be faced with the challenging task of justifying in terms of value, or contribution to the objectives of the firm, the existence of our present practices. For such a re-examination, let us be assured that we can find no comfort in pure objectivity under certainty. Furthermore, as indicated by recent trends toward managerial emphasis in accounting instruction, the accountant is viewed as an important member of the management team with policy-making duties. How can one make decisions that govern the future without resorting to subjective probabilities and value judgments?

It appears that the accountant is faced today with the same dilemma that faced economists in the postwar period. For years the economists have been trying to achieve the recognition that they could contribute to industry and the nation much more than "classroom dissemination of inapplicable knowledge." Finally the recognition came and for quite a while we saw the profession confused and bewildered as well as torn apart. On one side were those who wanted to rise up to the challenge and make value judgments with nationwide policy implications, while others kept on claiming that the function of the economist is to analyze consequences of actions affecting allocation of resources, and let others make the decisions. Those of you familiar with the literature on welfare economics will appreciate the economists' soul searching and tension that I have in mind. Nonetheless, so successful has been the transition in just a few years, that today there is the feeling in some circles that economists may be involved just a little too much in national policy decisions. Consequently, should the accountants now start running because new responsibilities appear to be thrust upon them as a recognition of what they have to offer? Frankly I do not think so and I hope that I am not in the minority.

What does this imply then for us in the academic field? It implies that not only do we have to learn to use these modern tools in our research on integrated system designs, but also that we must prepare our students to face the pending transition. As teachers, we should be looking forward to great challenges.

Still in the area of operational or optimizing tools one must mention the use of mathematical programming in optimizing the allocation of given *limited* resources, and determining the impact of product mix as well as establishing the factor mix that will minimize the cost of a given output.

As anyone can readily appreciate, unless there is some opportunity cost in the utilization of resources the mere use of sophisticated mathematical tools will not resolve anything. Often we do spend an enormous amount of effort and energy in devising and applying techniques that will result in exactness and purity, while using in a deterministic way input data that are very crude and not far from being the outcome of a random process of selection with unknown parameters. It is for this reason that I believe more efforts should be devoted in obtaining reliable and relevant data and interactions between variables, rather than manipulating crude information under certainty with the aid of sophisticated tools. Decision rules for repetitive operations can be perfected and programmed, but we cannot as yet program easily the processes that distinguish between relevant and nonrelevant information.

Admittedly a lot of the aforementioned uses of mathematical techniques may not fall into the area of traditional cost accounting, but frankly how can a modern controller avoid involvement in all aspects of control be these of financial, service, manufacturing or distribution origin?

Finally before going to the uses of mathematics in model building, I would like to mention the use of matrix algebra for cost allocations. This I do with certain misgivings, firstly because of the futility of most overhead cost distributions, and secondly because of the illogicality of stressing exactness in the allocation of a total that is so nebulous and depends upon choosing a basis of allocation that is fraught with arbitrariness. All of us are aware that the methods used for fixed overhead allocations are arbitrary. In using matrix algebra, the percentages of the costs that we feel should go to the various activities are used as inputs in the matrix that will be inverted for solving the simultaneous equations. Now do we really gain anything by such "exactness"? Definitely not! While there is no doubt that we preserve mathematical integrity in the application (programming) of imperfect decision rules, we must not lose sight of the value of the information that we so obtain. However, here it is for what it is worth. All those who believe in full cost and are not happy with the method of disposition of amounts that "properly" belong to overhead accounts that have already been closed out, and who do not wish to embark in secondary, tertiary and so *ad infinitum* redistributions, can use matrix algebra to get "exact" allocation of costs to production centers.

Use of mathematics in model building. In addition to the uses of mathematics as an operational or optimizing tool more recently efforts have been directed toward mathematical model building.[13] The purpose of models is to depict operations so as to study the interrelationships between variables (endogenous as well as exogenous), or the impact of inputs on outputs. Input coefficients as well as the powers used in mathematical models may be either real (statistically derived) or assumed, depending on what the investigator can easily obtain and what he wishes to study. For example one may assume ideal or potential interrelationships between variables in order to change behavior and determine the expected improvement that will be manifested if such changes are effected, or alternatively may be interested in studying the actual characteristics of a system (interactions between variables and interrelationships between inputs and outputs) in which case he may attempt to reproduce a "true" replica of operations as of a moment of time. This latter type of model building where the investigator generates an imitation of operations is related to simulation.

Mathematical simulation which has grown to maturity in the last few years has had a pronounced influence on the design of feedback-control systems. The mathematical knowledge that one may find extremely useful for such models includes in addition to differential and integral calculus, differential and difference equations.

Contrary to the belief that may be held by various people, mathematical models

[13] Examples of such efforts are:
C. W. Bastable, "Business Games, Models and Accounting," *Journal of Accountancy*, March 1960.
Philip L. Blumenthal, Jr., "Financial Models as Elementary Business Models," *N.A.A. Bulletin*, Vol. XLII, No. 8, April 1961, pp. 77–84.
Richard Mattessich, "Budgeting Models and System Simulation," The Accounting Review, Vol. XXXVI, No. 3, July 1961, pp. 384–97.
Richard Mattessich, "Mathematical Models in Business Accounting," The Accounting Review, Vol. XXXIII, No. 3, July 1958, pp. 472–81.
Allan B. Richards, "Input-Output Accounting for Business," The Accounting Review, Vol. XXXV, No. 3, July 1960, pp. 429–36.
Andrew C. Stedry, *Budget Control and Cost Behavior*, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1960.

depicting parts of the total operations of the firm are not overall optimizing. An inherent assumption behind partial optimizations is the existence of independence among subunits, which independence in turn implies that the results of partial optimizations are additive. In other words it is assumed that the total value function of the firm is linear homogeneous of zero order, in which case one could take the results of partial optimizations, find their least common multiple, and thus arrive at an optimum optimorum. Such assumptions of course are not true and often very damaging. It is well known that the subunits of the firm are interdependent and complementary, consequently optimizations cannot be made in isolation. The phrase "other things being equal," which the economists have so much popularized, does not provide any shelter or sanction for partial optimizations, but only a starting point for understanding the interrelationships that exist between interacting subunits. All this I stress not because I believe that partial optimizations are of no value, but as a justification for my preference of aggregative models, if such models can be obtained without leading to diffusion.

For quite some time now I have been intrigued by the similarities between vectors and the planning as well as operating activities of the firm. Decisions and operations like vectors have *intensity* and *direction*. Intensity indicates the degree of accomplishment or pursuit of each objective, and direction the channeling of effort toward certain areas or objectives to the possible exclusion of others. Motivated by these similarities and the belief that decisions tend to be additive rather than multiplicative, I proceeded to formulate a vector space model of the firm by applying to the planning as well as the operating activities of the firm the theorems of linear algebra. In the process I have identified the translation of objectives to subobjectives (both at the substantive planning and the operational levels) with mapping operations on vector spaces spanned by different bases, and naturally imposed the requirement that operations create an isomorphism with plans. In other words having broken down the substantive overall vector objective to vector subobjectives and then the latter to subsubobjectives and so on, and translated all these substantive plans into operational plans and activity budgets, then the next step was to assume that at each hierarchical level a comparison is made in order to determine whether there exists one to one correspondence between plans and operations.

By means of the above mentioned model one can view at once the total picture of the organizational and control structure of the firm with all its intricacies. Issues involving dynamic interactions between the various functional activities of the firm, interdependencies between operations, costs and time periods, all fall into a meaningful pattern.

I have introduced and fused the above mentioned vector space notions in our undergraduate industrial accounting and control course at the Massachusetts Institute of Technology,[14] and was very pleasantly surprised by the results. When explained in terms of vectors and mapping operations, notions such as economies of scale, indivisibilities, complementarities of resources at any moment of time and over time, interdependencies, economics of overhead costs, control structures, centralization vs decentralization and transfer pricing, appear to be easily understood. As an indication of what can be accomplished, I would like to mention that the instruc-

[14] By the time MIT students take such a course they have already had three semesters of Calculus, one of Differential Equations, and one of Probability and Statistics, but not necessarily any courses in Linear Algebra.

tion time was cut by at least one third when the new approach was introduced. Furthermore the students gain a much wider perspective of the total firm and its operations, which I believe is very vital in any efforts toward optimizations.

Without getting involved into detailed explanations, let me close by briefly summarizing some of the results that one can prove or deduce from such a model.

1. That the number of truly independent assignments or responsibility centers cannot be greater than the number of independent objectives. However within the context of a vector subspace (namely at a lower level in the hierarchy) one can by a mere choice of a basis create artificial independence which is often in practice mistaken as being real.[15]

2. The fewer the components in the vector objectives of a responsibility center the greater the degree of *decentralization from without* and the greater the authority for the exercise of value judgements that rests with the responsibility center. This, however, does not prejudice the degree of centralization or decentralization from within. One has to examine the translation of this activity vector to its components, in order to find out how much artificial independence is introduced.

3. Given economies of scale in specialization the fewer the components in the vector objective the greater the size of the responsibility center. Note that economies of scale are achieved through the introduction of inflexibilities. These can either take the form of time fixities or time interdependence of costs, or interdependence of operations given any short-run period.

4. The greater the number of translations of objectives to subobjectives (mappings) the greater the degree of interdependence that is introduced among all tasks and operations, but the less conscious each responsibility center becomes of such interdependence. Consequently the manifested need for information at low hierarchial levels is small.

5. The greater the conscious interdependence of tasks the greater the number of channels of communication and coordination that is necessary in order to assure that the proper information is available to the operatives to facilitate the accomplishment of their tasks. As a corollary to this we can say that the greater the conscious interdependence the greater the need for mutual interaction for the accomplishment of the common objective. If mutual interaction is not economically advantageous or feasible (the determination of the number of components of each vector objective is part of this issue) then proper accounting and other information must be supplied to the various activities so as to disclose to them what has transpired during the reporting interval in the outside world, and thus help them condition their expectations about the future. The length of the interval covered by these reports depends on the time sensitivity of the decisions of the various activities.

6. If we analyze the expected risk under mutual interaction, we will often find that hierarchial structures are necessary for the survival of complex dynamic organisms. Namely, there may exist a need for substability for survival. If nothing else hierarchy allows for short term task divisibility.[16]

7. To the extent that our activity vectors can be multi-dimensional, future budgets may differ drastically from what we know now and profit center organization and transfer pricing may be eliminated as means for inducing efficient allocation of resources.

8. Since the null space in which translation of objectives into subobjectives occurs can be of infinite dimension, *a priori* theorizing may give way to *a posteriori* learning.

In conclusion then we can say that mathematics is and can be of immense usefulness in accounting instruction and research. I only hope that by giving you such a fleeting preview of some of these uses, present as well as potential, that I have not done an injustice to such an important subject.

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[15] To bolster such artificial independence costly buffers or neutral zones may be created. Examples of such buffers are subassemblies and excess inventories at the various stages of manufacture.

[16] For more discussion on this issue see: Herbert A. Simon, *The New Sciences of Management Decision*, Harper and Brothers, New York, 1960, pp. 40–50.